Note: ***For the benefit of the students, specially the aspiring ones, the question of JEE(advanced), 2023 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test. For this test the time allocated in Mathematics, Physics and Chemistry are 30 minutes, 20 minutes and 25 minutes respectively.***

SOLUTIONS TO JEE (ADVANCED) – 2023 (PAPER-2)

MATHEMATICS

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

Q.1. Let $f : [1, \infty) \rightarrow \mathbb{R}$ be a differentiable function such that $f(1) = \frac{1}{3}$ and $3\int_1^x f(t)\,dt = x\,f(x) - \frac{x^3}{3}$, $x \in [1, \infty)$. Let e denote the base of the natural logarithm. Then the value of $f(e)$ is

(A) $\frac{e^2+4}{3}$

(B) $\frac{\log_e 4 + e}{3}$

(C) $\frac{4e^2}{3}$

(D) $\frac{e^2-4}{3}$

Sol. **C**

$$3\int_1^x f(x)\,dt = x\ f(x) - \frac{x^3}{3}$$

$\Rightarrow$ $3f(x) = f(x) + x f'(x) - x^2$ (using Newton's Leibniz theorem)

$\Rightarrow$ $\frac{dy}{dx} - \frac{2}{x}y = x$

I.F. = $e^{\int -\frac{2}{x}dx} = e^{-2\log x} = \frac{1}{x^2}$

$\Rightarrow$ solution is

$$\frac{y}{x^2} = \log_e x + c$$

$$y = x^2 \log_e x + cx^2$$

But $f(1) = \frac{1}{3}$

$\Rightarrow \quad \frac{1}{3} = c$

$\Rightarrow \quad y = x^2 \log_e x + \frac{1}{3}x^2$

Now $y(e) = e^2 + \frac{1}{3}e^2 = \frac{4}{3}e^2$

Q.2. Consider an experiment of tossing a coin repeatedly until the outcomes of two consecutive tosses are same. If the probability of a random toss resulting in head is $\frac{1}{3}$, then the probability that the experiment stops with head is

(A) $\frac{1}{3}$
(B) $\frac{5}{21}$
(C) $\frac{4}{21}$
(D) $\frac{2}{7}$

Sol. **B**

$P(H) = \frac{1}{3},\ P(T) = \frac{2}{3}$

P = P(H H) + {P(T H H) + P(T H T H H) + P(T H T H T H H) + … ∞)
+ {P(H T H H) + P(H T H T H H) + P(H T H T H T H H) + .. ∞)

$$= \frac{1}{9} + \left(\frac{2}{3}\times\frac{1}{9} + \frac{2}{3}\times\frac{1}{3}\times\frac{2}{3}\times\frac{1}{9} + ...\infty\right) + \left(\frac{1}{3}\times\frac{2}{3}\times\frac{1}{9} + \frac{1}{3}\times\frac{2}{3}\times\frac{1}{3}\times\frac{2}{3}\times\frac{1}{9} + ...\infty\right)$$

$$= \frac{1}{9} + \frac{\frac{2}{27}}{1-\frac{2}{9}} + \frac{\frac{1}{3}\times\frac{2}{3}\times\frac{1}{9}}{1-\frac{2}{9}}$$

$$= \frac{1}{9} + \frac{2}{3\times 7} + \frac{2}{7\times 9} = \frac{5}{21}.$$

Q.3. For any $y \in \mathbb{R}$, let $\cot^{-1}(y) \in (0, \pi)$ and $\tan^{-1}(y) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the sum of all the solutions of the equation $\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \cot^{-1}\left(\frac{9-y^2}{6y}\right) = \frac{2\pi}{3}$ for $0 < |y| < 3$, is equal to

(A) $2\sqrt{3} - 3$
(B) $3 - 2\sqrt{3}$
(C) $4\sqrt{3} - 6$
(D) $6 - 4\sqrt{3}$

Sol. **C**

If $0 < y < 3$ then given equation can be written as

$$\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \tan^{-1}\left(\frac{6y}{9-y^2}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{6y}{9-y^2}\right) = \frac{\pi}{3}$$

$$\Rightarrow \quad \tan\left(\tan^{-1}\frac{6y}{9-y^2}\right) = \tan\frac{\pi}{3}$$

$$\Rightarrow \quad \frac{6y}{9-y^2} = \sqrt{3} \Rightarrow \sqrt{3}y^2 + 6y - 9\sqrt{3} = 0$$

$$\Rightarrow \quad y = \frac{-6+\sqrt{36+108}}{2\sqrt{3}} = \frac{6}{2\sqrt{3}} = \sqrt{3}$$

If – 3 < y < 0 then given equation can be written as

$$\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \tan^{-1}\left(\frac{6y}{9-y^2}\right) + \pi = \frac{2\pi}{3}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{6y}{9-y^2}\right) = -\frac{\pi}{6}$$

$$\Rightarrow \quad \tan\left(\tan^{-1}\left(\frac{6y}{9-y^2}\right)\right) = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad 6\sqrt{3}y = -9 + y^2 \Rightarrow y^2 - 6\sqrt{3}y - 9 = 0$$

$$\Rightarrow \quad y = \frac{6\sqrt{3}-\sqrt{144}}{2} = \frac{6\sqrt{3}-12}{2} = 3\sqrt{3}-6$$

Therefore sum of all the solutions

$$= 3\sqrt{3} - 6 + \sqrt{3} = 4\sqrt{3} - 6.$$

Q.4. Let the position vectors of the points *P, Q, R* and S be $\vec{a} = \hat{i} + 2\hat{j} - 5\hat{k}, \vec{b} = 3\hat{i} + 6\hat{j} + 3\hat{k}$, $\vec{c} = \frac{17}{5}\hat{i} + \frac{16}{5}\hat{j} + 7\hat{k}$ and $\vec{d} = 2\hat{i} + \hat{j} + \hat{k}$, respectively. Then which of the following statements is true?

(A) The points *P, Q, R* and S are **NOT** coplanar

(B) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides *PR* internally in the ratio 5 : 4

(C) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides *PR* externally in the ratio 5 : 4

(D) The square of the magnitude of the vector $\vec{b} \times \vec{d}$ is 95

Sol. **B**

$\vec{a} = \hat{i} + 2\hat{j} - 5\hat{k}$, $\vec{b} = 3\hat{i} + 6\hat{j} + 3\hat{k}$, $\vec{c} = \frac{17}{5}\hat{i} + \frac{16}{5}\hat{j} + 7\hat{k}$, $\vec{d} = 2\hat{i} + \hat{j} + \hat{k}$

$\overrightarrow{PQ} = 2\hat{i} + 4\hat{j} + 8\hat{k}$, $\overrightarrow{PR} = \frac{12}{5}\hat{i} + \frac{6}{5}\hat{j} + 12\hat{k}$, $\overrightarrow{PS} = \hat{i} - \hat{j} + 6\hat{k}$

$$\left[\overrightarrow{PQ}\ \overrightarrow{PR}\ \overrightarrow{PS}\right] = \frac{1}{5}\begin{bmatrix} 2 & 4 & 8 \\ 12 & 6 & 60 \\ 1 & -1 & 6 \end{bmatrix} = 0$$

A is incorrect

$$\left|\vec{b} \times \vec{d}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 6 & 3 \\ 2 & 1 & 1 \end{vmatrix} = 3\hat{i} + 3\hat{j} - 9\hat{k}$$

$$\left|\vec{b} \times \vec{d}\right|^2 = 9 + 9 + 81 = 99$$

D option is incorrect

$$\frac{\vec{b}+2\vec{d}}{3}=\frac{7\hat{i}+8\hat{j}+5\hat{k}}{3}=\frac{21\hat{i}+24\hat{j}+15\hat{k}}{9}$$

$$\frac{5\vec{c}+4\vec{a}}{9}=\frac{\vec{b}+2\vec{d}}{3}$$

B is correct.

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : –2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get –2 marks.

Q.5. Let M = (a_{ij}), $i, j \in \{1, 2, 3\}$, be the 3 × 3 matrix such that $a_{ij} = 1$ if $j + 1$ is divisible by i, otherwise $aij = 0$. Then which of the following statements is(are) true?

(A) M is invertible

(B) There exists a nonzero column matrix $\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$ and such that $M\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}=\begin{pmatrix}-a_1\\-a_2\\-a_3\end{pmatrix}$

(C) The set $\{X \in \mathbb{R}^3 : MX = 0\} \neq \{\mathbf{0}\}$, where $\mathbf{0}=\begin{pmatrix}0\\0\\0\end{pmatrix}$

(D) The matrix (M – 2I) is invertible, where I is the 3 × 3 identity matrix

Sol. **B, C**

$$M=\begin{bmatrix}1&1&1\\1&0&1\\0&1&0\end{bmatrix}$$

$\because$ |M| = 0 so matrix is non-invertible so option A is incorrect

$$\begin{bmatrix}1 & 1 & 1\\1 & 0 & 1\\0 & 1 & 0\end{bmatrix}\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}=\begin{bmatrix}a_1+a_2+a_3\\a_1+a_3\\a_2\end{bmatrix}=\begin{bmatrix}-a_1\\-a_2\\-a_3\end{bmatrix}$$

$2a_1 + a_2 + a_3 = 0$, $a_2 + a_1 + a_3 = 0$ $a_2 + a_3 = 0$, $a_1 = 0 \Rightarrow a_2 = -a_3$

So B is correct

$a_1 + a_2 + a_3 = 0$, $a_1 + a_3 = 0$, $a_2 = 0 \Rightarrow a_1 = -a_3$

So C is correct

$$|M-2I| = \begin{vmatrix}-1 & 1 & 1\\1 & -2 & 1\\0 & 1 & -2\end{vmatrix}$$

$= (-1)3 - (-2) + 1 \times 1$

$= -3 + 2 + 1 = 0$

So M – 2I is non-invertible.

Q.6. Let $f : (0, 1) \to \mathbb{R}$ be the function defined as $f(x) = [4x]\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right)$, where $[x]$ denotes the greatest integer less than or equal to x . Then which of the following statements is(are) true?

(A) The function f is discontinuous exactly at one point in (0, 1)

(B) There is exactly one point in (0, 1) at which the function f is continuous but **NOT** differentiable

(C) The function f is **NOT** differentiable at more than three points in (0, 1)

(D) The minimum value of the function f is $-\frac{1}{512}$

Sol. **A, B**

$$f(x) = [4x]\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right)$$

$$f(x) = \begin{cases} 0 & 0 < x < \frac{1}{4} \\ 1\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right) & \frac{1}{4} \le x < \frac{1}{2} \\ 2\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right) & \frac{1}{2} \le x < \frac{3}{4} \\ 3\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right) & \frac{3}{4} \le x < 1 \end{cases}$$

$f\left(\frac{3^-}{4}\right) = \frac{1}{8}$; $f\left(\frac{3^+}{4}\right) = \frac{3}{16}$ so f(x) is discontinuous at $x = \frac{3}{4}$. So, **(A)** option is correct

$$f'(x) = \begin{cases} 0 & ; \quad 0 < x < \frac{1}{4} \\ 4\left(x-\frac{1}{4}\right)^2 + 2\left(x-\frac{1}{4}\right)\left(x-\frac{1}{2}\right) & ; \quad \frac{1}{4} \le x < \frac{1}{2} \\ 4\left(x-\frac{1}{4}\right)\left(x-\frac{1}{2}\right) + 2\left(x-\frac{1}{4}\right)^2 & ; \quad \frac{1}{2} \le x < \frac{3}{4} \\ 3\left(x-\frac{1}{4}\right)^2 + 6\left(x-\frac{1}{4}\right)\left(x-\frac{1}{2}\right) & ; \quad \frac{3}{4} \le x < 1 \end{cases}$$

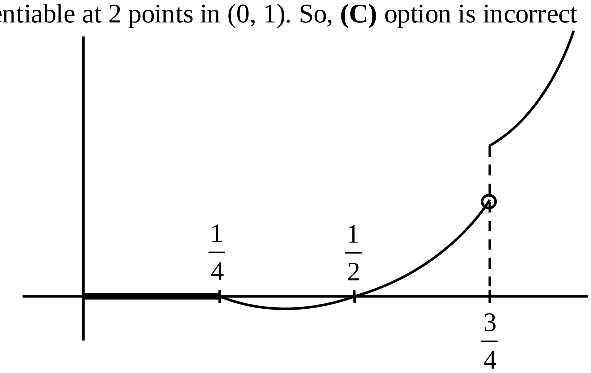
$f'\left(\frac{1^-}{4}\right) = 0$, $f'\left(\frac{1^+}{4}\right) = 0$ so differentiable at $x = \frac{1}{4}$

$f'\left(\frac{1^-}{2}\right) = \frac{1}{16}$, $f'\left(\frac{1^+}{2}\right) = 2\cdot\frac{1}{16} = \frac{1}{8}$ so non-differentiable at $x = \frac{1}{2}$. So, **(B)** option is correct

$f'\left(\frac{3^-}{4}\right) = 4\cdot\frac{1}{2}\cdot\frac{1}{4} + 2\cdot\frac{1}{4} = 1$

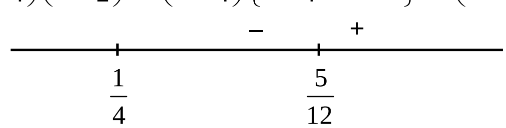
$f'\left(\frac{3^+}{4}\right) = 3\cdot\frac{1}{4} + 6\cdot\frac{1}{2}\times\frac{1}{4} = \frac{3}{2}$ so non-differentiable at $\frac{3}{4}$

$\because$ f(x) is non-differentiable at 2 points in (0, 1). So, **(C)** option is incorrect



$f(x) = \left(x - \frac{1}{4}\right)^2\left(x - \frac{1}{2}\right); \frac{1}{4} \le x < \frac{1}{2}$

$f'(x) = \left(x - \frac{1}{4}\right)^2 + 2\left(x - \frac{1}{4}\right)\left(x - \frac{1}{2}\right) = \left(x - \frac{1}{4}\right)\left\{x - \frac{1}{4} + 2x - 1\right\} = \left(x - \frac{1}{4}\right)\left(3x - \frac{5}{4}\right)$



$f_{min} = f\left(\frac{5}{12}\right) = \left(\frac{5}{12} - \frac{1}{4}\right)^2\left(\frac{5}{12} - \frac{1}{2}\right) = -\frac{1}{12}\times\frac{1}{36} = -\frac{1}{432}$. So, **(D)** option is wrong

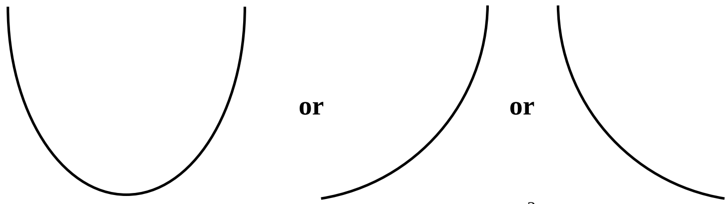
Q.7. Let *S* be the set of all twice differentiable functions *f* from $\mathbb{R}$ to $\mathbb{R}$ such that $\frac{d^2 f}{dx^2}(x) > 0$ for all $x \in (-1, 1)$. For $f \in S$, let X_f be the number of points $x \in (-1, 1)$ for which $f(x) = x$. Then which of the following statements is(are) true?

(A) There exists a function $f \in S$ such that $X_f = 0$
(B) For every function $f \in S$, we have $X_f \le 2$
(C) There exists a function $f \in S$ such that $X_f = 2$
(D) There does **NOT** exist any function f in S such that $X_f = 1$

Sol. **A, B, C**

$\frac{d^2f}{dx^2} > 0 \Rightarrow$ f(x) is concave upward in (–1, 1)

Graph of y = f(x) must be



The line y = x can cut it in 0, 1, or 2 points (A) holds for $f(x) = x^2 + 1$
(C) holds for $f(x) = 2x^2$ and number of points of intersection will be ≤ 2

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;
Zero Marks : 0 In all other cases.

Q.8. For $x \in \mathbb{R}$, let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the minimum value of the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \int_0^{x\tan^{-1}x} \frac{e^{(t-\cos t)}}{1+t^{2023}} dt$ is

Sol. **0**

$$f(x) = \int_0^{x\tan^{-1}x} \frac{e^{(t-\cos t)}}{1+t^{2023}} dt$$

$$f'(x) = \frac{e^{x\tan^{-1}x - \cos(x\tan^{-1}x)}}{1+(x\tan^{-1}x)^{2023}} \left[\frac{x}{1+x^2} + \tan^{-1}x\right] = 0$$

$\Rightarrow \quad x + \tan^{-1}x(1+x^2) = 0$, x = 0

Minimum value of f(x) = 0

Q.9. For $x \in \mathbb{R}$, let $y(x)$ be a solution of the differential equation $(x^2-5)\frac{dy}{dx} - 2xy = -2x(x^2-5)^2$ such that $y(2) = 7$. Then the maximum value of the function y(x) is

Sol. **16**

$$(x^2-5)\frac{dy}{dx} - 2xy = -2x(x^2-5)^2$$

$$\frac{dy}{dx} - \frac{2x}{x^2-5}y = -2x(x^2-5)$$

$$\text{I.F.} = e^{\int -\frac{2x}{x^2-5}dx} = e^{-\ln(x^2-5)}$$

$= \frac{1}{x^2 - 5}$

$y \cdot \frac{1}{x^2 - 5} = \int -2x \, dx$

$y \cdot \frac{1}{x^2 - 5} = -x^2 + c$

$-7 = -4 + c \Rightarrow c = -3$

Now $y = -(x^4 - 2x^2 - 15)$

$y = -((x^2 - 1)^2 - 16)$

so maximum value = 16

Q.10. Let X be the set of all five digit numbers formed using 1, 2, 2, 2, 4, 4, 0. For example, 22240 is in X while 02244 and 44422 are not in X. Suppose that each element of X has an equal chance of being chosen. Let p be the conditional probability that an element chosen at random is a multiple of 20 given that it is a multiple of 5. Then the value of $38p$ is equal to

Sol. **31**

0, 1, 2, 2, 2, 4, 4

Number divisible by 5 (event A)

= Coefficient of x^4 in $4!\left(x^0 + x^1\right)\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)\left(1 + x + \frac{x^2}{2!}\right)$ = 38

Number divisible by 20 must end in 20 or 40 = 31 (event B)

$P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right) = P(B)P\left(\frac{A}{B}\right)$

$\Rightarrow P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{31}{38}$ $\therefore$ 38p = 31

*Q.11. Let $A_1, A_2, A_3, \ldots, A_8$ be the vertices of a regular octagon that lie on a circle of radius 2. Let P be a point on the circle and let PA_i denote the distance between the points P and A_i for i =1, 2, ..., 8. If P varies over the circle, then the maximum value of the product $PA_1 \cdot PA_2 \ldots PA_8$, is

Sol. **512**

Let P be $2e^{i\theta}$

$PA_k = \left|2e^{i\theta} - 2e^{i\frac{2k\pi}{8}}\right|$

$\prod_{k=1}^{8} PA_k = 2^8 \cdot 2^8 \prod_{k=1}^{8} \sin\left(\frac{\theta}{2} + \frac{k\pi}{8}\right) = \frac{2^{16}}{2^7}\sin(4\theta)$

Maximum $\prod_{k=1}^{8} PA_k = 2^9$

Q.12. Let $R = \left\{ \begin{pmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{pmatrix} : a, b, c, d \in \{0, 3, 5, 7, 11, 13, 17, 19\} \right\}$. Then the number of invertible matrices in R is

Sol. **3780**

Total matrices = 8^4 = 4096

|R| = 5(bc – ad)

No. of non-invertible matrices:

bc = ad

Case I: if a, b, c, d $\neq$ 0, then cases = ${}^{7}C_2 \,.\, 2!\, 2! + {}^{7}C_1(1) = 91$

Case II: if ad = bc = 0, then cases = ${}^{15}C_1 \,.\, {}^{15}C_1 = 225$

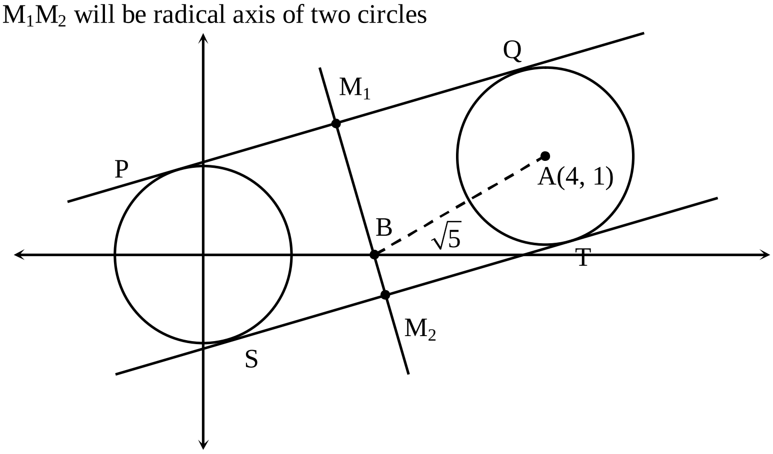
(ad & bc can take any combination from $0 \times 0, 0 \times 3, 0 \times 5, \ldots, 0 \times 19, 3 \times 0, 5 \times 0, \ldots 19 \times 0$)

No. of invertible matrices = 4096 – (91 + 225) = 3780

*Q.13. Let C_1 be the circle of radius 1 with center at the origin. Let C_2 be the circle of radius r with center at the point $A = (4, 1)$, where $1 < r < 3$. Two distinct common tangents PQ and ST of C_1 and C_2 are drawn. The tangent PQ touches C_1 at P and C_2 at Q. The tangent ST touches C_1 at S and C_2 at T. Mid points of the line segments PQ and ST are joined to form a line which meets the x-axis at a point B. If $AB = \sqrt{5}$, then the value of r^2 is

Sol. **2**

Line joining M_1M_2 will be radical axis of two circles



$C_1 : x^2 + y^2 - 1 = 0$

$C_2 : (x-4)^2 + (y-1)^2 = r^2$

$x^2 + y^2 - 8x - 2y + (17 - r^2) = 0$

Line M_1M_2: $(x^2 + y^2 - 1) - (x^2 + y^2 - 8x - 2y + (17 - r^2)) = 0$

$8x + 2y + (r^2 - 18) = 0$

$$\text{Point } B = \left(\frac{18 - r^2}{8}, 0\right)$$

$$AB = \sqrt{5}$$

$$\sqrt{\left(\frac{18-r^2}{8} - 4\right)^2 + (0-1)^2} = \sqrt{5}$$

$$\left(\frac{r^2 + 14}{8}\right)^2 = 4$$

$$\frac{r^2 + 14}{8} = 2 \Rightarrow r^2 = 2$$

SECTION 4 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;

  Zero Marks : 0 In all other cases.

PARAGRAPH "I"

Consider an obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

*Q.14. Let *a* be the area of the triangle *ABC*. Then the value of $(64a)^2$ is

PARAGRAPH "I"

Consider an obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

*Q.15. Then the inradius of the triangle *ABC* is

Sol. Let A > B > C

$A - C = \frac{\pi}{2}$

a + c = 2b

R = 1

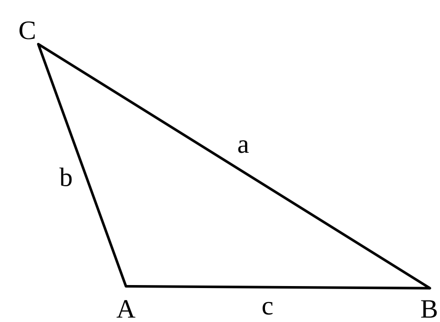
A + B + C = π

$\left(\frac{\pi}{2} + C\right) + B + C = \pi$

$\Rightarrow B + 2C = \frac{\pi}{2}$

2R sin A + 2R sin C = 2(2R sin B)

sin C + cos C = 2 cos 2C



$\cos C - \sin C = \frac{1}{2}$

$\sin C = \frac{-1+\sqrt{7}}{4}$ only

$\sin A = \frac{\sqrt{7}+1}{4}$

$\sin B = \frac{\sqrt{7}}{4}$

14. **1008**

Area of $\Delta ABC = 2R^2 \sin A \sin B \sin C$

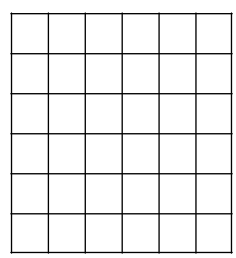
$a = \frac{2}{64}\left(6\sqrt{7}\right) \Rightarrow (64\,a)^2 = 1008$

15. **0.25**

$r = \frac{\Delta}{s} = 0.25$

PARAGRAPH "II"

Consider the 6 × 6 square in the figure. Let $A_1, A_2, \ldots\ldots, A_{49}$ be the points of intersections (dots in the picture) in some order. We say that A_i and A_j are friends if they are adjacent along a row or along a column. Assume that each point A_i has an equal chance of being chosen.



(There are two questions based on PARAGRAPH "II", the question given below is one of them)

Q.16. Let p_i be the probability that a randomly chosen point has i many friends, i = 0, 1, 2, 3, 4. Let X be a random variable such that for i = 0, 1, 2, 3, 4, the probability $P(X = i) = p_i$. Then the value of $7E(X)$ is

Sol. **24**

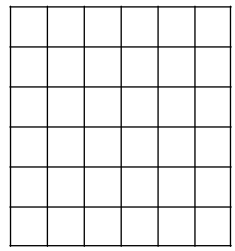
x	P(x)	x . P(x)
0	P(0) = 0	0
1	P(1) = 0	0
2	$P(2) = \frac{4}{49}$	$\frac{8}{49}$
3	$P(3) = \frac{20}{49}$	$\frac{60}{49}$
4	$P(4) = \frac{25}{49}$	$\frac{100}{49}$

$E(x) = E \times P(x) = \frac{168}{49} = \frac{24}{7}$

7E(x) = 24.

PARAGRAPH "II"

Consider the 6×6 square in the figure. Let $A_1, A_2, \ldots, A_{49}$ be the points of intersections (dots in the picture) in some order. We say that A_i and A_j are friends if they are adjacent along a row or along a column. Assume that each point A_i has an equal chance of being chosen.



(There are two questions based on PARAGRAPH "II", the question given below is one of them)

Q.17. Two distinct points are chosen randomly out of the points $A_1, A_2, \ldots, A_{49}$. Let p be the probability that they are friends. Then the value of $7p$ is

Sol. **0.5**

2 Consecutive points can be chosen in $2 \times 6 \times 7$ ways = 84 ways

So, n(E) = 84 ; n(S) = $^{49}C_2$

So, $7p = 7 \times \frac{84}{^{49}C_2} = 0.5$

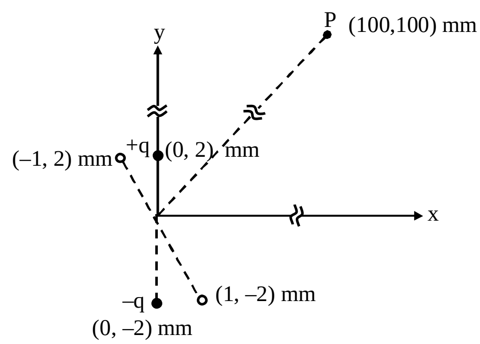
PHYSICS

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

Q.1 An electric dipole is formed by two charges +q and – q located in xy-plane at (0, 2) mm and (0, –2) mm, respectively, as shown in the figure. The electric potential at point P (100, 100) mm due to the dipole is V_0. The charges +q and –q are then moved to the points (–1, 2) mm and (1, –2) mm, respectively. What is the value of electric potential at P due to the new dipole?

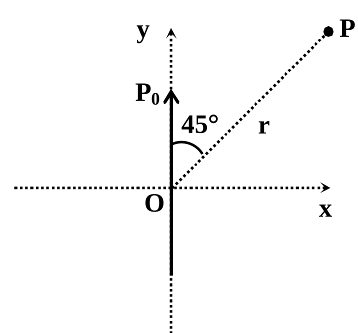


(A) $V_0/4$ (B) $V_0/2$ (C) $V_0/\sqrt{2}$ (D) $3V_0/4$

Sol. **B**

Case (i)

$$V_0 = \frac{kP_0}{r^2}\cos 45°$$



Case (ii)

Component of P_0' along x-axis = $-\frac{P_0}{2}\hat{i}$

Component of P_0' along y-axis = $P_0\hat{j}$

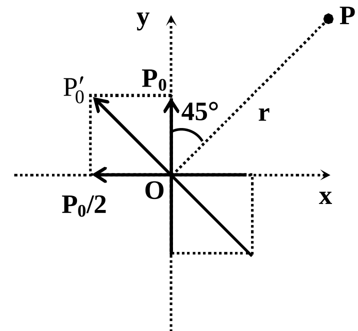
Potential due to $P_0\hat{j}$

$$\Rightarrow \frac{kP_0\cos 45°}{r^2} = V_0$$

Potential due to $-\frac{P_0}{2}\hat{i}$

$$\Rightarrow \quad \frac{k\left(\frac{P_0}{2}\right)\cos 135^\circ}{r^2} = -\frac{V_0}{2}$$

So, net potential due to new dipole $(P_0') = V_0 - \frac{V_0}{2} = \frac{V_0}{2}$



Q.2 Young's modulus of elasticity Y is expressed in terms of three derived quantities, namely, the gravitational constant G, Planck's constant h and the speed of light c, as $Y = c^\alpha h^\beta G^\gamma$. Which of the following is the correct option?

(A) $\alpha = 7, \beta = -1, \gamma = -2$
(B) $\alpha = -7, \beta = -1, \gamma = -2$
(C) $\alpha = 7, \beta = -1, \gamma = 2$
(D) $\alpha = -7, \beta = 1, \gamma = -2$

Sol. **A**

$Y = c^\alpha h^\beta G^\gamma$

$Y \to$ Young's modulus of elasticity
$c \to$ speed of light
$h \to$ plank's constant
$G \to$ Gravitational constant

$\left[ML^{-1}T^{-2}\right] = \left[LT^{-1}\right]^\alpha \left[ML^2T^{-1}\right]^\beta \left[M^{-1}L^3T^{-2}\right]^\gamma$

$\beta - \gamma = 1$ ...(i)
$\alpha + 2\beta + 3\gamma = -1$ ...(ii)
$-\alpha - \beta - 2\gamma = -2$
$\alpha + \beta + 2\gamma = 2$ ...(iii)

From (i), (ii) and (iii)
$\alpha = 7$, $\beta = -1$, $\gamma = -2$

*Q.3 A particle of mass m is moving in the xy-plane such that its velocity at a point (x, y) is given as $\vec{v} = \alpha\left(y\hat{x} + 2x\hat{y}\right)$, where α is a non-zero constant. What is the force $\vec{F}$ acting on the particle?

(A) $\vec{F} = 2m\alpha^2\left(x\hat{x} + y\hat{y}\right)$
(B) $\vec{F} = m\alpha^2\left(y\hat{x} + 2x\hat{y}\right)$
(C) $\vec{F} = 2m\alpha^2\left(y\hat{x} + x\hat{y}\right)$
(D) $\vec{F} = m\alpha^2\left(x\hat{x} + 2y\hat{y}\right)$

Sol. **A**

$\vec{v} = \alpha(y\hat{x} + 2x\hat{y})$

$\vec{a} = \alpha\left(\frac{dy}{dt}\hat{x} + 2\frac{dx}{dt}\hat{y}\right)$

$\vec{a} = \alpha\frac{dy}{dt}\hat{x} + 2\alpha\frac{dx}{dt}\hat{y}$ ...(i)

$\frac{dx}{dt} = \alpha y$ ...(ii)

$\frac{dy}{dt} = 2\alpha x$ ...(iii)

From (i), (ii) and (iii)

$\vec{a} = 2\alpha^2 x\hat{x} + 2\alpha^2 y\hat{y}$

$\vec{F} = m\vec{a}$

$\vec{F} = 2m\alpha^2(x\hat{x} + y\hat{y})$

*Q.4 An ideal gas is in thermodynamic equilibrium. The number of degrees of freedom of a molecule of the gas is n. The internal energy of one mole of the gas is U_n and the speed of sound in the gas is v_n. At a fixed temperature and pressure, which of the following is the correct option ?

(A) $v_3 < v_6$ and $U_3 > U_6$
(B) $v_5 > v_3$ and $U_3 > U_5$
(C) $v_5 > v_7$ and $U_5 < U_7$
(D) $v_6 < v_7$ and $U_6 < U_7$

Sol. **C**

$$U_n = \frac{n}{2}RT \quad \ldots(i)$$

$U_n \propto n$, where n is degree of freedom

As n → increases, hence; U_n → increases

$$V_n = \sqrt{\frac{\left(1+\frac{2}{n}\right)RT}{M}} \quad \ldots(ii)$$

where n is degree of freedom

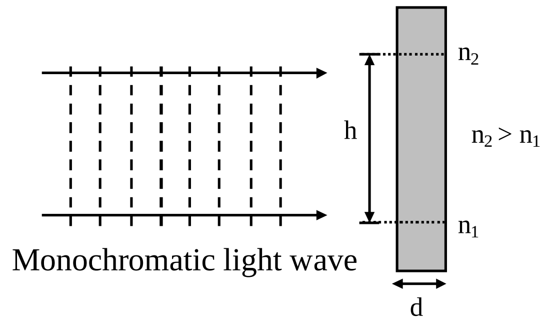
As n → increases, hence; V_n → decreases

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : +2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get –2 marks.

Q.5 A monochromatic light wave is incident normally on a glass slab of thickness d, as shown in the figure. The refractive index of the slab increases linearly from n_1 to n_2 over the height h. Which of the following statement(s) is(are) true about the light wave emerging out of the slab?



(A) It will deflect up by an angle $\tan^{-1}\left[\frac{\left(n_2^2 - n_1^2\right)d}{2h}\right]$.

(B) It will deflect up by an angle $\tan^{-1}\left[\frac{\left(n_2 - n_1\right)d}{h}\right]$.

(C) It will not deflect.

(D) The deflection angle depends only on $(n_2 - n_1)$ and not on the individual values of n_1 and n_2.

Sol. **B, D**

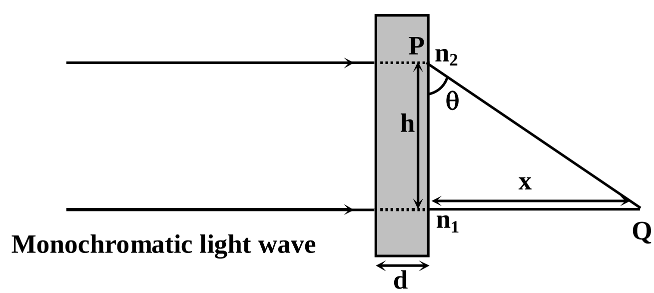
Wavefront PQ at time 't'

$$\frac{n_2 d}{c} = \frac{n_1 d}{c} + \frac{x}{c}$$

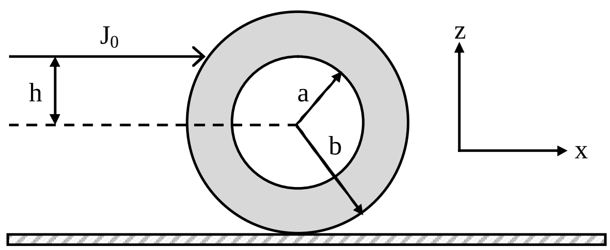
$$x = d(n_2 - n_1)$$

$$\tan\theta = \frac{x}{h} = \frac{(n_2 - n_1)d}{h}$$

$$\theta = \tan^{-1}\left[\frac{(n_2 - n_1)d}{h}\right]$$



*Q.6 An annular disk of mass M, inner radius a and outer radius b is placed on a horizontal surface with coefficient of friction μ, as shown in the figure. At some time, an impulse $J_0\hat{x}$ is applied at a height h above the center of the disk. If $h = h_m$ then the disk rolls without slipping along the x-axis. Which of the following statement(s) is(are) correct?



(A) For $\mu \neq 0$ and $a \to 0$, $h_m = b/2$

(B) For $\mu \neq 0$ and $a \to b$, $h_m = b$

(C) For $h = h_m$, the initial angular velocity does **not** depend on the inner radius a.

(D) For $\mu = 0$ and $h = 0$, the wheel always slides without rolling.

Sol. **A, B, C, D**

$J = Mv_{cm}$ …(i)

$Jh = I_{cm}\omega$ …(ii)

$v_{cm} = b\omega$ …(iii)

$h = \frac{I_{cm}}{Mb}$

(A) For $\mu \neq 0$, a = 0 the system will be a disc, for pure rolling of disc

$h = \frac{I_{cm}}{Mb} = \frac{b}{2}$

(B) for $\mu \neq 0$, a = b wheel will be ring

$h = b$

(C) for $\mu = 0$ and h = 0 wheel will slide without rolling

(D) for $h = h_m$, $v_{cm} = v\omega$

Q.7 The electric field associated with an electromagnetic wave propagating in a dielectric medium is given by $\vec{E} = 30(2\hat{x} + \hat{y})\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right] \text{V m}^{-1}$. Which of the following option(s) is(are) correct ?

[Given: The speed of light in vacuum, $c = 3 \times 10^8 \text{ m s}^{-1}$]

(A) $B_x = -2\times10^{-7}\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right] \text{Wb m}^{-2}$.

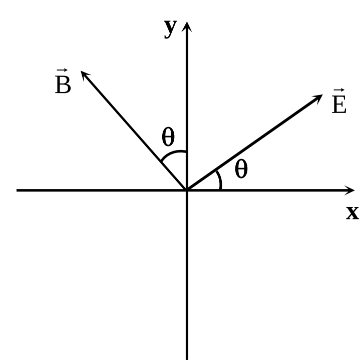
(B) $B_y = 2\times10^{-7}\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right] \text{Wb m}^{-2}$.

(C) The wave is polarized in the xy-plane with polarization angle 30° with respect to the x-axis.

(D) The refractive index of the medium is 2.

Sol. **A, D**

Speed of light in dielectric medium = $\frac{5\times10^{14}}{\left(\frac{10^7}{3}\right)} = 1.5\times10^8$ m/s



$\therefore$ refractive index = 2

E = BC

$30\sqrt{5} = B(1.5\times10^8)$

$B = 20\sqrt{5}\times10^{-8}$

$B_x = B\sin\theta = 2\times10^{-7}$

$B_y = B\cos\theta = 4\times10^{-7}$

For the given equation of EMW for electric field, magnetic field can be expressed as

$$B_x = -2\times10^{-7}\sin\left[2\pi(5\times10^{14}t - \frac{10^7}{3}z\right]\text{Wbm}^{-2}$$

$$B_y = 4\times10^{-7}\sin\left[2\pi(5\times10^{14}t - \frac{10^{-7}}{3}z\right]\text{Wbm}^{-2}$$

The wave is polarized in xy-plane with polarization angle $\tan^{-1}\left(\frac{1}{2}\right)$ with respect to x-axis

SECTION 3 (Maximum Marks: 24)

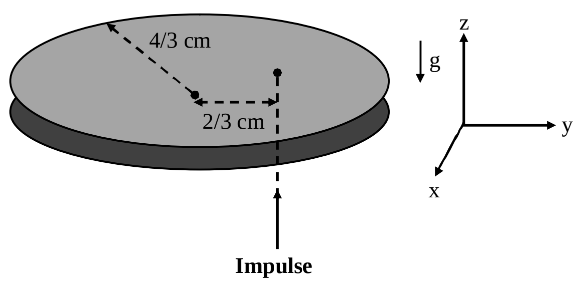
- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases.

*Q.8 A thin circular coin of mass 5 gm and radius 4/3 cm is initially in a horizontal xy –plane. The coin is tossed vertically up (+z direction) by applying an impulse of $\sqrt{\frac{\pi}{2}} \times 10^{-2}$ N-s at a distance 2/3 cm from its center. The coin spins about its diameter and moves along the +z direction. By the time the coin reaches back to its initial position, it completes *n* rotations. The value of n is ________.

[Given: The acceleration due to gravity g = 10 m s^{-2}]



Sol. **30**

$J = MV_{cm}$ . . . (1)

$J.\frac{R}{2} = \frac{MR^2}{4}\omega$ …(2)

From (1) $V_{cm} = J/M$

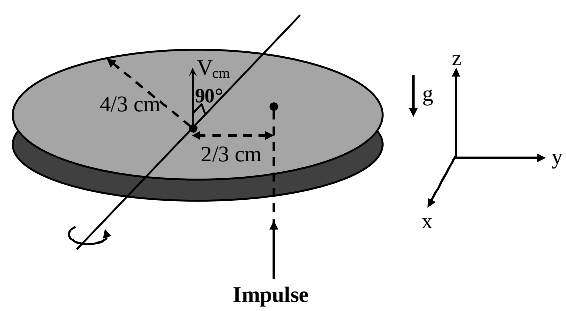
Time when coin reaches back to its initial position is $T = \frac{2V_{cm}}{g} = \frac{2J}{gM}$.

Angle rotated in time T is

$\theta = \omega t = \frac{2J}{MR}.\frac{2J}{Mg}$ [From (2)]

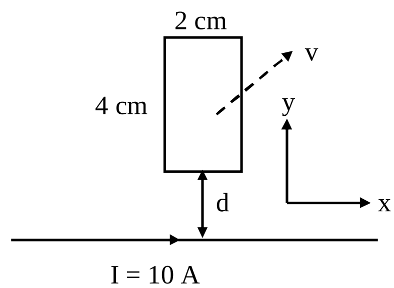
$\Rightarrow \theta = 60\,\pi$

$\therefore n = \frac{\theta}{2\pi} = \frac{60\pi}{2\pi} = 30$



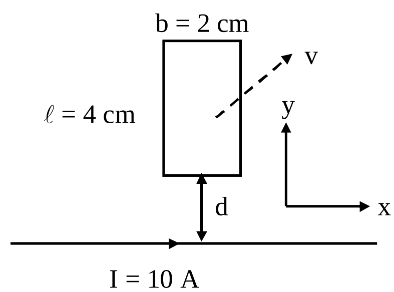
Q.9 A rectangular conducting loop of length 4 cm and width 2 cm is in the xy-plane, as shown in the figure. It is being moved away from a thin and long conducting wire along the direction $\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y}$ with a constant speed v. The wire is carrying a steady current *I* = 10 A in the positive x-direction. A current of 10 μA flows through the loop when it is at a distance d = 4 cm from the wire. If the resistance of the loop is 0.1 Ω, then the value of v is _______ m s^{-1}.

[Given: The permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2}]



Sol. **4**

Emf induced in the loop is



$\varepsilon = \frac{\mu_0 I}{2\pi d}.\frac{v}{2}b - \frac{\mu_0 I}{2\pi(d+\ell)}.\frac{v}{2}.b$

(ℓ = 4 cm, b = 2 cm, d = 4 cm)

$\Rightarrow \varepsilon = \frac{\mu_0 Ivb}{4\pi}\left(\frac{1}{d} - \frac{1}{d+\ell}\right)$

$\Rightarrow I_0 = \frac{\varepsilon}{R} = \frac{\mu_0 Ivb\ell}{4\pi d(d+\ell)R}$

Substituting all the values,
v = 4 m/s

*Q.10 A string of length 1 m and mass 2 × 10^{-5} kg is under tension T. When the string vibrates, two successive harmonics are found to occur at frequencies 750 Hz and 1000 Hz. The value of tension T is _____ Newton.

Sol. **5**

$\frac{nv}{2\ell} = 750\text{Hz}$

$\frac{(n+1)v}{2\ell} = 1000\text{Hz}$

$\therefore \frac{v}{2\ell} = 1000 - 750 = 250\text{Hz}$

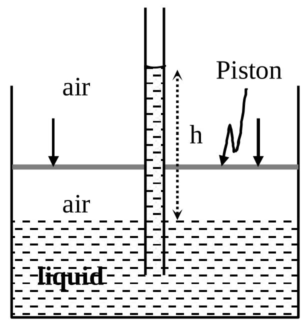
$\Rightarrow \quad v = 2 \times 250 \times 1$

$\Rightarrow \sqrt{\frac{T}{\mu}} = 500$

$\therefore \quad T = 5N.$

*Q.11 An incompressible liquid is kept in a container having a weightless piston with a hole. A capillary tube of inner radius 0.1 mm is dipped vertically into the liquid through the airtight piston hole, as shown in the figure. The air in the container is isothermally compressed from its original volume V_0 to $\frac{100}{101}V_0$ with the movable piston. Considering air as an ideal gas, the height (h) of the liquid column in the capillary above the liquid level in cm is_______.

[Given: Surface tension of the liquid is 0.075 N m^{-1}, atmospheric pressure is 10^5 N m^{-2}, acceleration due to gravity (g) is 10 m s^{-2}, density of the liquid is 10^3 kg m^{-3} and contact angle of capillary surface with the liquid is zero]



Sol. **25**

$P_0V_0 = P_A\left(\frac{100}{101}V_0\right)$

$P_A = P_0\left(\frac{101}{100}\right)$

$\because\ P_A = P_D$

$\Rightarrow\quad P_D = \left(\frac{101}{100}\right)P_0$

Also, $P_B - P_C = \frac{2T}{r}$

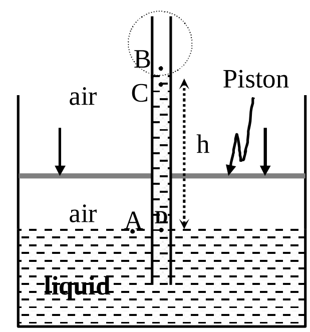
$P_C = P_B - \frac{2T}{r} = P_0 - \frac{2T}{r}$ (since, $P_B = P_0$)

$P_D = \left(\frac{101}{100}\right)P_0 = P_C + \rho gh$

$\Rightarrow\quad \left(\frac{101}{100}\right)P_0 = \left(P_0 - \frac{2T}{r}\right) + \rho gh$

Solving we get,

h = 0.25 m = 25 cm.



Q.12 In a radioactive decay process, the activity is defined as $A = -\frac{dN}{dt}$, where N(t) is the number of radioactive nuclei at time t. Two radioactive sources, S_1 and S_2 have same activity at time $t = 0$. At a later time, the activities of S_1 and S_2 are A_1 and A_2, respectively. When S_1 and S_2 have just completed their 3^{rd} and 7^{th} half-lives, respectively, the ratio A_1/A_2 is __________.

Sol. **16**

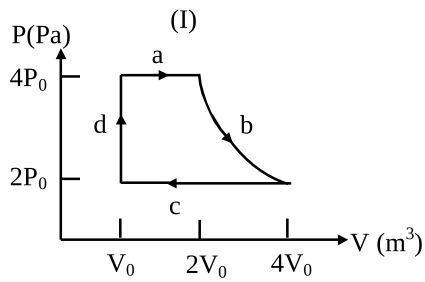
After 3^{rd} half lives, $A_1 = A_0(1/2)^3$

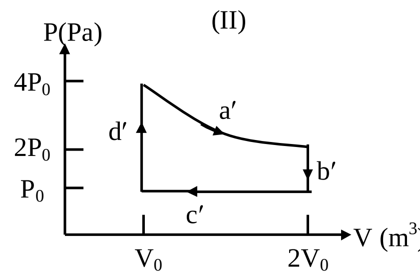
After 7^{th} half lives, $A_2 = A_0(1/2)^7$

Where A_0 = initial activity.

$\left(\frac{A_1}{A_2}\right) = 16$

*Q.13 One mole of an ideal gas undergoes two different cyclic processes I and II, as shown in the P-V diagrams below. In cycle I, processes a, b, c and d are isobaric, isothermal, isobaric and isochoric, respectively. In cycle II, processes a′, b′, c′ and d′ are isothermal, isochoric, isobaric and isochoric, respectively. The total work done during cycle I is W_I and that during cycle II is W_{II}. The ratio W_I / W_{II} is _______.





Sol. **2**

Work done in cycle I is

$W_I = 4P_0V_0 + 8P_0V_0\ \ell n2 - 6P_0V_0$

$W_{II} = 4P_0V_0\ell n2 - P_0V_0$

$\frac{W_I}{W_{II}} = 2$.

SECTION 4 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

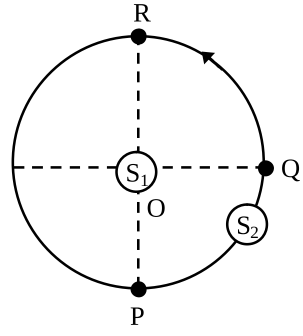
Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;

Zero Marks : 0 In all other cases.

PARAGRAPH I

S_1 and S_2 are two identical sound sources of frequency 656 Hz. The source S_1 is located at O and S_2 moves anti-clockwise with a uniform speed $4\sqrt{2}$ m s^{-1} on a circular path around O, as shown in the figure. There are three points P, Q and R on this path such that P and R are diametrically opposite while Q is equidistant from them. A sound detector is placed at point *P*. The source S_1 can move along direction OP.

[Given: The speed of sound in air is 324 m s^{-1}]



*Q.14 When only S_2 is emitting sound and it is at Q, the frequency of sound measured by the detector in Hz is _________.

Sol. **648**

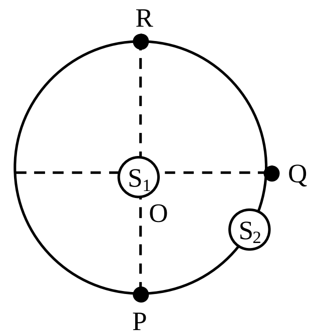
Apparent frequency,

$$f = \left(\frac{v}{v + v_s \cos 45}\right) f_0$$

$$f = \left(\frac{324}{324+4}\right) 656 \qquad (v_s = 4\sqrt{2}\ m/s)$$

$$f = \left(\frac{324}{328} \times 656\right)$$

f = 648 Hz.



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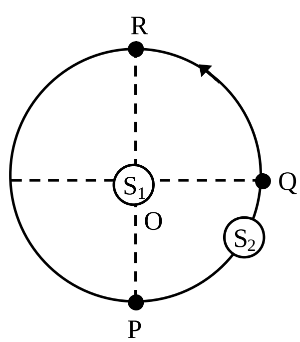


*Q.15 Consider both sources emitting sound. When S_2 is at R and S_1 approaches the detector with a speed 4 m s^{-1}, the beat frequency measured by the detector is _______Hz.

Sol. **8.2**

$$f_1 = \left(\frac{v}{v - v_s'}\right) f_0 \qquad \left(v_s' = 4 m/s\right)$$

$$f_1 = \left(\frac{324}{324-4}\right) 656 = \frac{324}{320} \times 656$$

f_1 = 664.2 Hz

Now,

$$f_2 = \left(\frac{v}{v - v_s \cos 90}\right) f_0 = f_0$$

f_2 = 656 Hz.

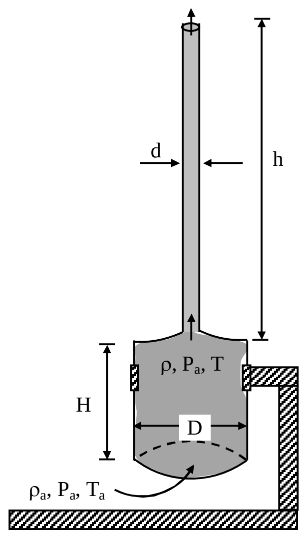
Hence, the beats frequency measured by the detector.

$f_b = |f_1 - f_2|$ = 664.2 – 656 = 8.2 Hz.

PARAGRAPH II

A cylindrical furnace has height (H) and diameter (D) both 1 m. It is maintained at temperature 360 K. The air gets heated inside the furnace at constant pressure P_α and its temperature becomes T = 360 K. The hot air with density ρ rises up a vertical chimney of diameter d = 0.1 m and height h = 9 m above the furnace and exits the chimney (see the figure). As a result, atmospheric air of density $\rho_\alpha = 1.2$ kg m^{-3}, pressure P_α and temperature $T_\alpha = 300$ K enters the furnace. Assume air as an ideal gas, neglect the variations in ρ and T inside the chimney and the furnace. Also ignore the viscous effects.

[Given: The acceleration due to gravity g = 10 m s^{-2} and $\pi = 3.14$]



*Q.16 Considering the air flow to be streamline, the steady mass flow rate of air exiting the chimney is _______ gm s^{-1}.

Sol. **49.61**

P_a = constant

So, $\rho_a T_A = \rho T$

So, $\rho = \dfrac{\rho_0 T_a}{T} = \dfrac{1.2 \times 300}{360}$

$\rho = 1$ kg/m^3

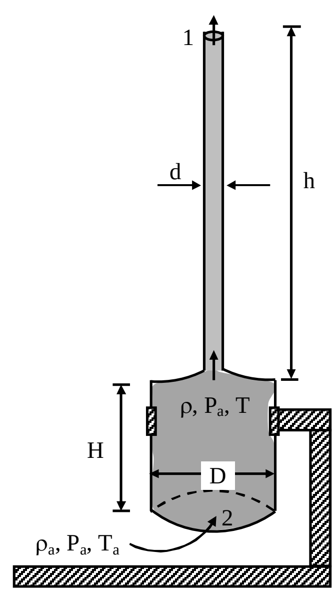
Applying Bernoulli's equation between points (1) and (2)

Assuming velocity of hot air inside furnace ~ 0

$$P_a + 0 + 0 = P_a - \rho_a g(10) + \rho g(10) + \frac{1}{2}\rho v^2$$

$$\text{So, } v = \sqrt{\frac{2(\rho_a - \rho)g(10)}{\rho}} = \sqrt{(2)(0.2)100} = \sqrt{40}\ \text{m/s}$$

$$Q = \frac{\pi(0.1)^2}{4}(\sqrt{40})\,(10^3) = 49.61\ \text{gm/s}$$



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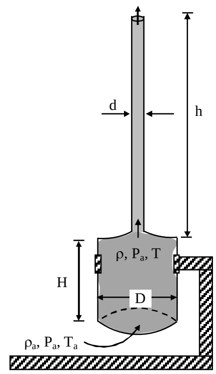


*Q.17 When the chimney is closed using a cap at the top, a pressure difference ΔP develops between the top and the bottom surfaces of the cap. If the changes in the temperature and density of the hot air, due to the stoppage of air flow, are negligible then the value of ΔP is ______ N m^{-2}.

Sol. **20**

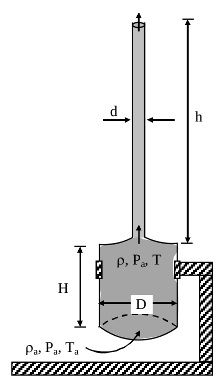
$P_a = P_{inside} + \rho g\ (10)$

$P_{inside} = P_a - \rho g\ (10)$

$P_{outside} = P_a - \rho_a g(10)$

$\Delta P = P_{inside} - P_{outside} = (\rho_a - \rho)g \times 10$

$= 20\ N/m^2$



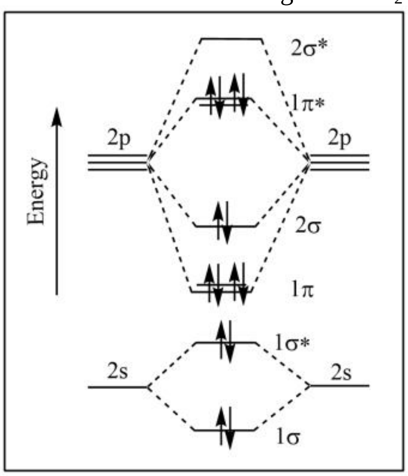
CHEMISTRY

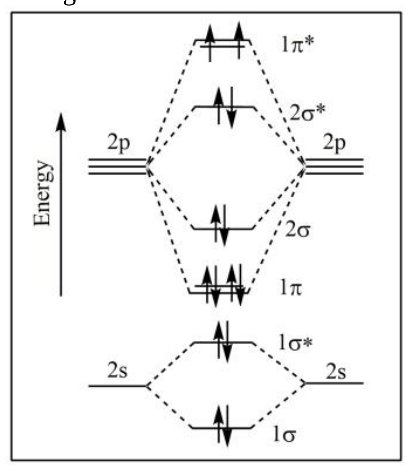
SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the
- correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

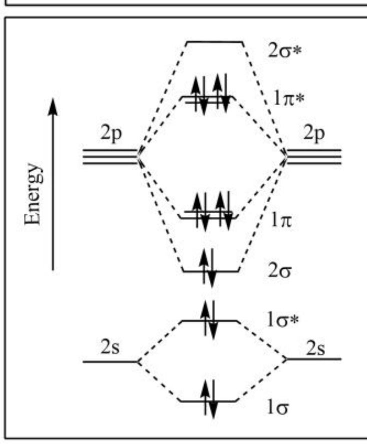
*Q. 1 The correct molecular orbital diagram for F_2 molecule in the ground state is

(A)

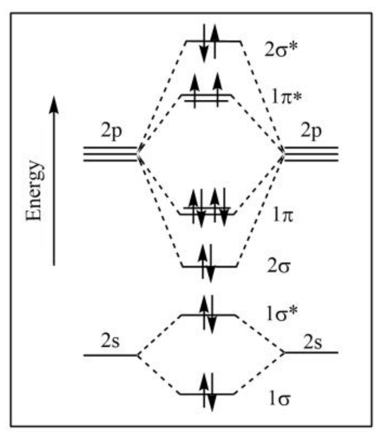




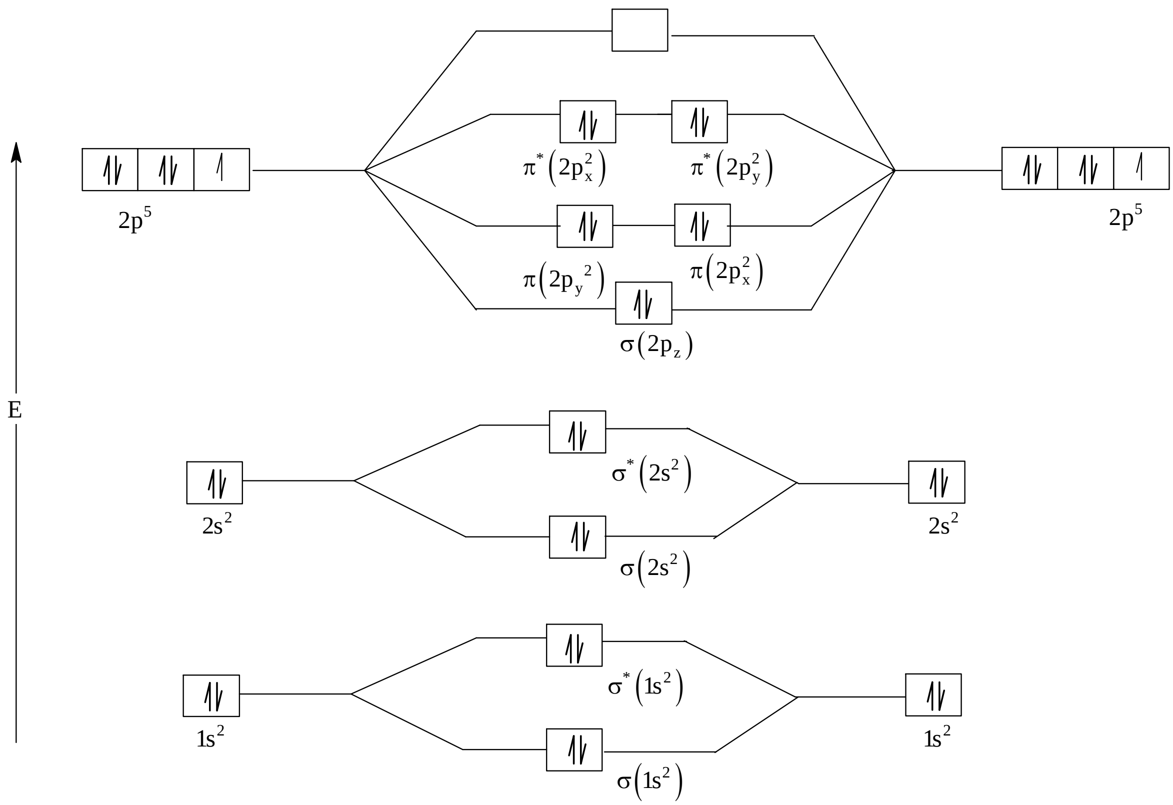
(C)



(D)



Sol. **C**



Q. 2 Consider the following statements related to colloids.

(I) Lyophobic colloids are **not** formed by simple mixing of dispersed phase and dispersion medium.

(II) For emulsions, both the dispersed phase and the dispersion medium are liquid.

(III) Micelles are produced by dissolving a surfactant in any solvent at any temperature.

(IV) Tyndall effect can be observed from a colloidal solution with dispersed phase having the same refractive index as that of the dispersion medium.

The option with the correct set of statements is

(A) (I) and (II) (B) (II) and (III)

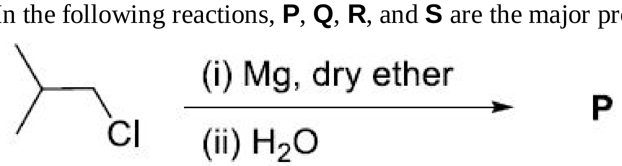
(C) (III) and (IV) (D) (II) and (IV)

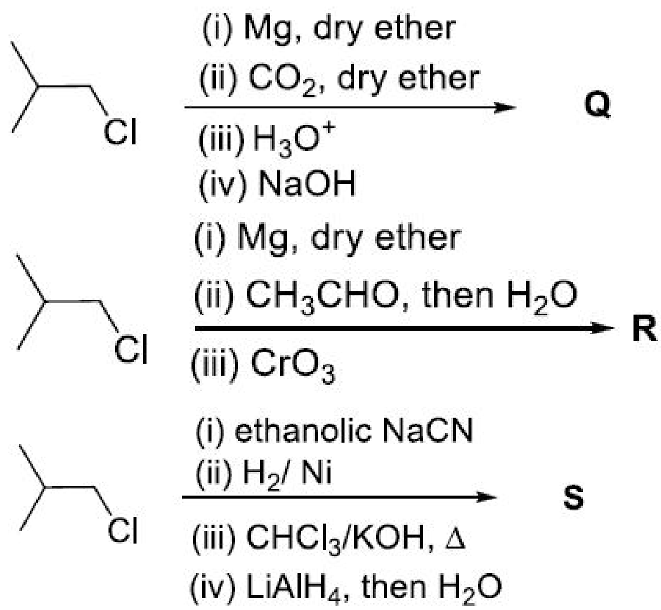
Sol. **A**

In tyndall effect, refractive indices of dispersed phase and dispersion medium differ greatly in magnitude.

- Micelles are formed by surfactant at CMC or above CMC and at Kraft temperature or above Kraft temperature.

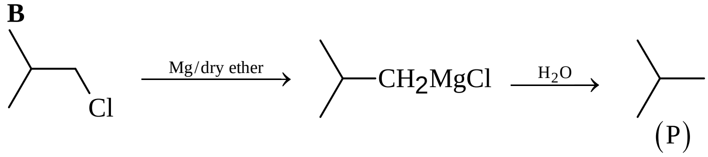
Q. 3 In the following reactions, **P**, **Q**, **R**, and **S** are the major products.

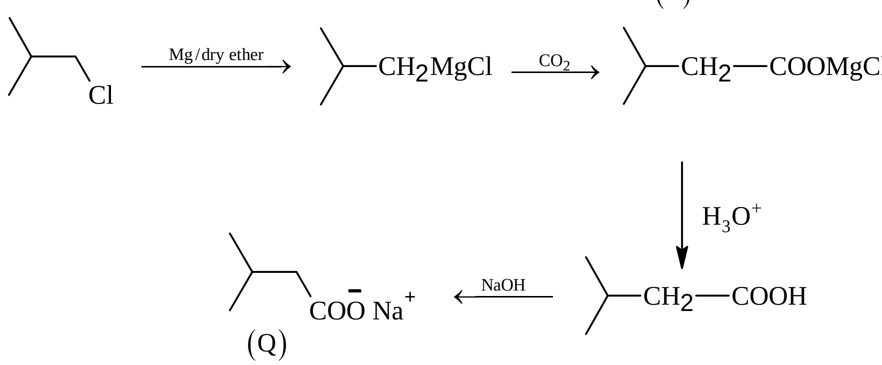


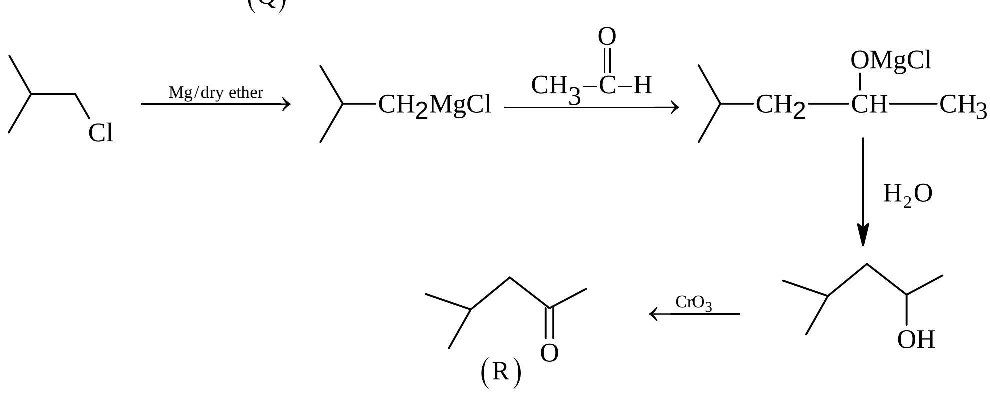


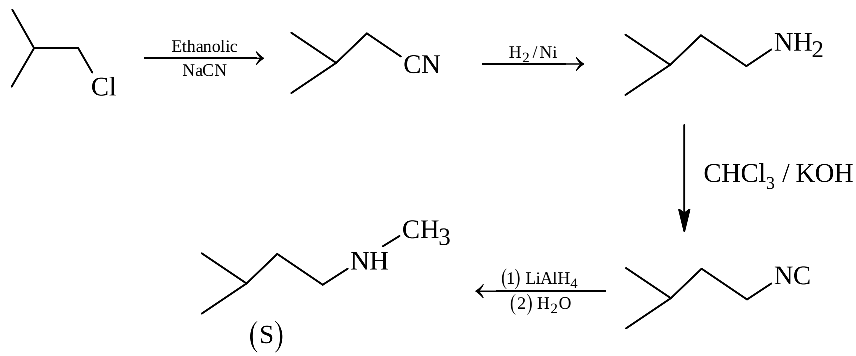
The correct statement about **P**, **Q**, **R**, and **S** is
(A) **P** is a primary alcohol with four carbons.
(B) **Q** undergoes Kolbe's electrolysis to give an eight-carbon product.
(C) **R** has six carbons and it undergoes Cannizzaro reaction.
(D) **S** is a primary amine with six carbons.

Sol. **B**



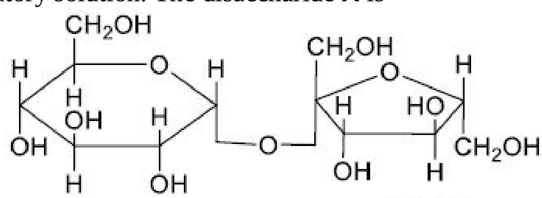




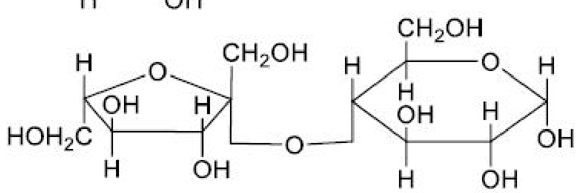


Q.4 A disaccharide **X** cannot be oxidised by bromine water. The acid hydrolysis of **X** leads to a laevorotatory solution. The disaccharide **X** is

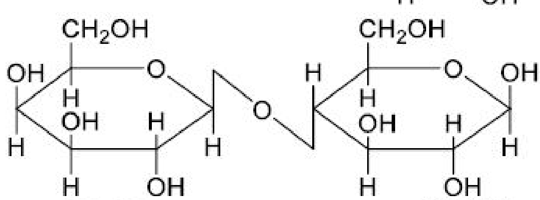
(A)



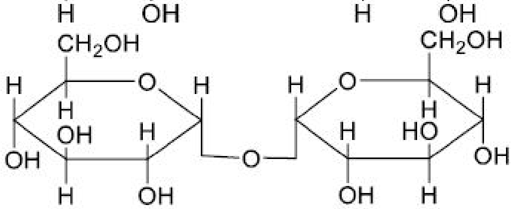
(B)



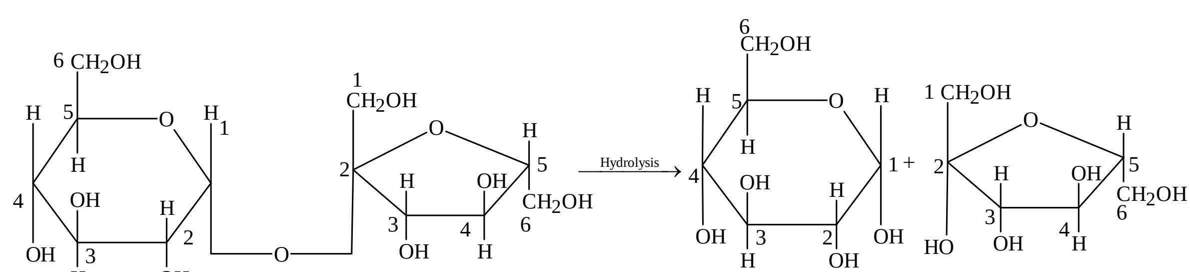
(C)



(D)



Sol. **A**



$\alpha-D(+)$Glucopyranose $\beta-D(+)$Fructofuranose

Sucrose
(dextrorotatory)

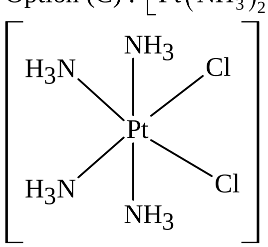
Hydrolysis of sucrose brings about a change in the sign of rotation from dextro(+) to laevo(–) and the product named as invert sugar.

SECTION 2 (Maximum Marks: 12)

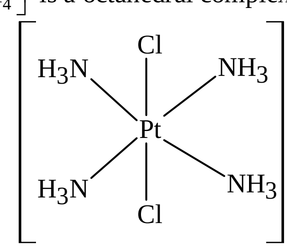
- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
- *Negative Marks :* −2 In all other cases. · For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get −2 marks.

Q.5 The complex(es), which can exhibit the type of isomerism shown by $[Pt(NH_3)_2Br_2]$, is(are)
$[en = H_2NCH_2CH_2NH_2]$
(A) $[Pt(en)(SCN)_2]$ (B) $[Zn(NH_3)_2Cl_2]$
(C) $[Pt(NH_3)_2Cl_4]$ (D) $[Cr(en)_2(H_2O)(SO_4)]^+$

Sol. **C, D**
$\left[Pt\left(NH_3\right)_2 Br_2\right]$ is a square planar complex.
The given compound can show geometrical isomerism (cis-trans form)
Option (A) : $\left[Pt(en)(SCN)_2\right]$ cannot show geometrical isomerism.
Option (B) : $\left[Zn\left(NH_3\right)_2 Cl_2\right]$ is a tetrahedral complex, cannot show geometrical isomerism.
Option (C) : $\left[Pt\left(NH_3\right)_2 Cl_4\right]$ is a octahedral complex, can show geometrical isomerism.

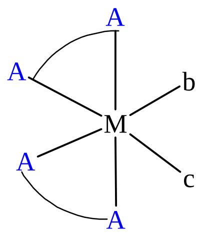


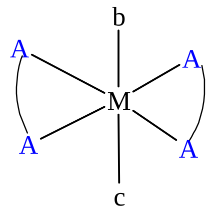
cis – form



trans – form

Option (D) : $\left[Cr(en)_2\left(H_2O\right)\left(SO_4\right)\right]^+$ is octahedral complex and is of type $\left[M(AA)_2 bc\right]$, can show geometrical isomerism.





Q.6 Atoms of metals x, y, and z form face-centred cubic (fcc) unit cell of edge length L_x, body-centred cubic (bcc) unit cell of edge length L_y, and simple cubic unit cell of edge length L_z, respectively.

If $r_z = \frac{\sqrt{3}}{2} r_y ; r_y = \frac{8}{\sqrt{3}} r_x ; M_z = \frac{3}{2} M_y$ and $M_Z = 3M_x$.

then the correct statement(s) is(are)

[Given: M_x, M_y, and M_z are molar masses of metals x, y, and z, respectively.

r_x, r_y, and r_z are atomic radii of metals x, y, and z, respectively.]

(A) Packing efficiency of unit cell of x > Packing efficiency of unit cell of y > Packing efficiency of unit cell of z

(B) $L_y > L_z$

(C) $L_x > L_y$

(D) Density of x > Density of y

Sol. **A, B, D**

For metal 'x'

Fcc: Edge length , $a_1 = L_x$

For metal 'y'

Bcc: Edge length , $a_2 = L_y$

For metal 'z'

Bcc: Edge length , $a_3 = L_z$

$$r_z = \frac{\sqrt{3}}{2} r_y, r_y = \frac{8}{\sqrt{3}} r_x, M_z = \frac{3}{2} M_y \text{ and } M_Z = 3M_x$$

For option (A)

(i) For FCC (Z = 4) metal 'x', $4r_x = \sqrt{2} L_x$

$$P.E = \frac{Z \times \frac{4}{3}\pi (r_x)^3}{a_1^3} = \frac{4 \times \frac{4}{3}\pi (r_x)^3}{(L_x)^3} = \frac{4 \times \frac{4}{3}\pi (r_x)^3}{\left(\frac{4}{\sqrt{2}} r_x\right)^3} = 0.24\pi$$

(ii) For BCC (Z = 2) metal 'y', $4r_y = \sqrt{3} L_y$

$$P.E = \frac{Z \times \frac{4}{3}\pi (r_y)^3}{a_2^3} = \frac{2 \times \frac{4}{3}\pi (r_y)^3}{(L_y)^3} = \frac{2 \times \frac{4}{3}\pi (r_y)^3}{\left(\frac{4}{\sqrt{3}} r_y\right)^3} = 0.22\pi$$

(iii) For SC (Z = 1) metal 'z', $2r_z = L_z$

$$P.E = \frac{Z \times \frac{4}{3}\pi (r_z)^3}{a_3^3} = \frac{1 \times \frac{4}{3}\pi (r_z)^3}{(L_z)^3} = \frac{1 \times \frac{4}{3}\pi (r_z)^3}{(2r_z)^3} = \frac{\pi}{6} = 0.17\pi$$

$$(P.E)_{FCC} > (P.E)_{BCC} > (P.E)_{SC}$$

For option (B)

$$4r_y = \sqrt{3}L_y \qquad 2r_z = L_z$$

$$L_y = \frac{4r_y}{\sqrt{3}}$$

$$\frac{L_y}{L_z} = \frac{4r_y}{\sqrt{3}\times 2r_z} = \frac{2r_y}{\sqrt{3}r_z} = \frac{2r_y}{\sqrt{3}.\frac{\sqrt{3}}{2}r_y} = \frac{4}{3}$$

So, $L_y > L_z$

For option (C)

$$4r_x = \sqrt{2}L_x, \ 4r_y = \sqrt{3}L_y$$

$$L_x = \frac{4r_x}{\sqrt{2}}, \ L_y = \frac{4r_y}{\sqrt{3}}$$

$$\frac{L_x}{L_y} = \frac{\sqrt{3}r_x}{\sqrt{2}\times 8/\sqrt{3}r_x} = \frac{3}{8\sqrt{2}}$$

So, $L_x < L_y$ **incorrect**

For option (D)

$$d_x = \frac{4\times M_x}{\left(\frac{4r_x}{\sqrt{2}}\right)^3 \times N_A}$$

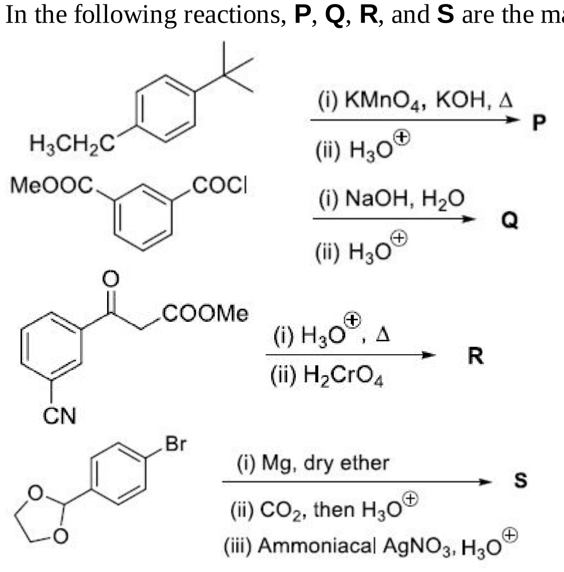
$$d_y = \frac{2\times M_y}{\left(\frac{4r_y}{\sqrt{3}}\right)^3 \times N_A}$$

$$r_y = \frac{8}{\sqrt{3}}r_x, \ \frac{M_x}{M_y} = \frac{1}{2}$$

$$\frac{d_x}{d_y} = \frac{512}{2\sqrt{2}} = \frac{256}{\sqrt{2}}$$

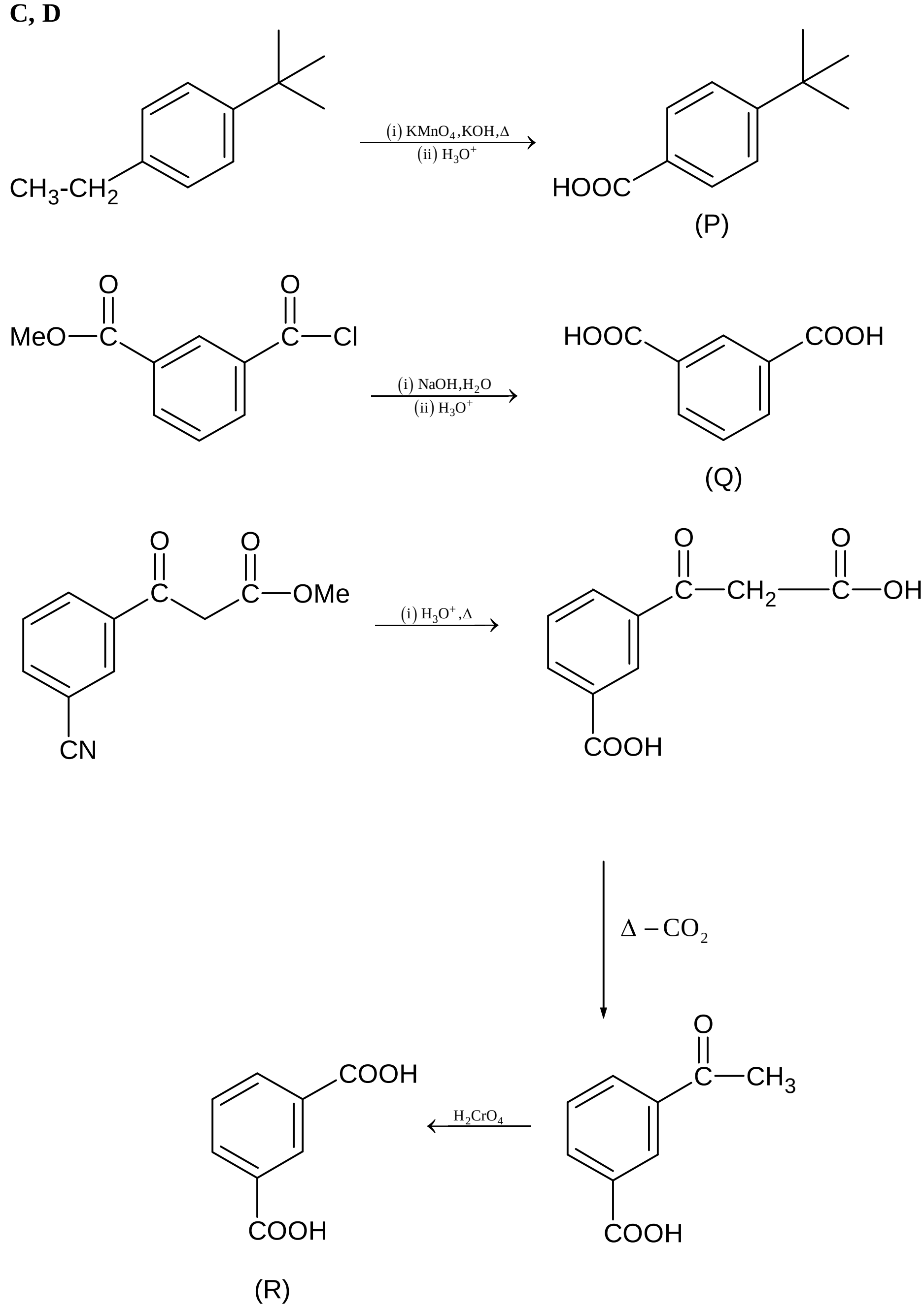
So $d_x > d_y$ (correct)

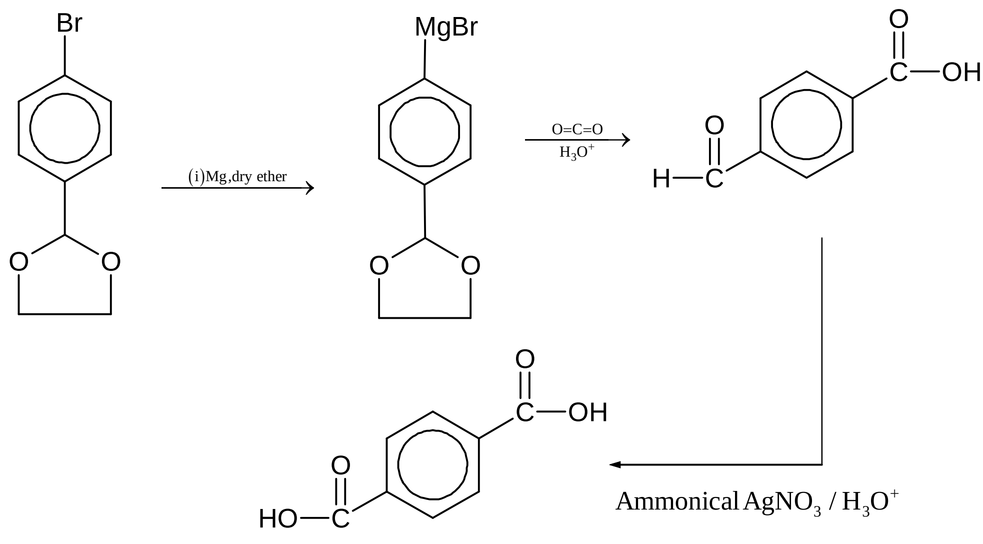
Q.7 In the following reactions, **P**, **Q**, **R**, and **S** are the major products.



The correct statement(s) about **P**, **Q**, **R**, and **S** is(are)
(A) **P** and **Q** are monomers of polymers dacron and glyptal, respectively.
(B) **P**, **Q**, and **R** are dicarboxylic acids.
(C) Compounds **Q** and **R** are the same.
(D) **R** does **not** undergo aldol condensation and **S** does **not** undergo Cannizzaro reaction.

Sol. **C, D**





(S)

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 In all other cases.

*Q.8 H_2S (5 moles) reacts completely with acidified aqueous potassium permanganate solution. In this reaction, the number of moles of water produced is **x**, and the number of moles of electrons involved is **y**. The value of (**x** + **y**) is ____.

Sol. **18**

$$\left(8H^+ + MnO_4^- + 5e^- \longrightarrow Mn^{2+} + 4H_2O\right) \times 2$$

$$\left(H_2S \longrightarrow S + 2H^+ + 2e^-\right) \times 5$$

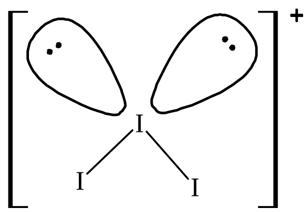
$$2MnO_4^- + 16H^+ + 5H_2S \longrightarrow 2Mn^{2+} + 5S + 10H^+ + 8H_2O$$

$$2MnO_4^- + 6H^+ + 5H_2S \longrightarrow 2Mn^{2+} + 5S + 8H_2O$$

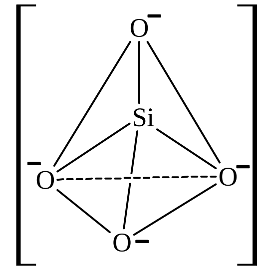
x = 8
y = 10

Q.9 Among $[I_3]^+$, $[SiO_4]^{4-}$, SO_2Cl_2, XeF_2, SF_4, ClF_3, $Ni(CO)_4$, XeO_2F_2, $[PtCl_4]^{2-}$, XeF_4, and $SOCl_2$, the total number of species having sp^3 hybridised central atom is ______.

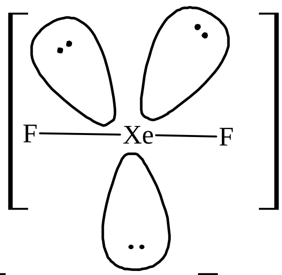
Sol. **5**

I_3^+


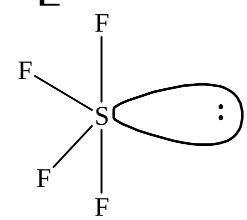
$[sp^3]$

$[SiO_4]^{4-}$


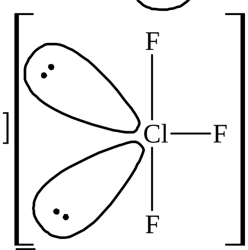
$[sp^3]$

$[XeF_2]$


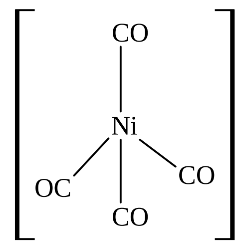
$[sp^3d]$

$[SF_4]$


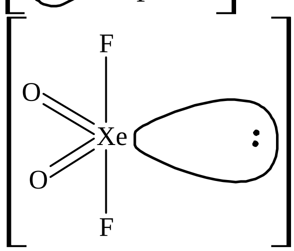
$[sp^3d]$

$[ClF_3]$


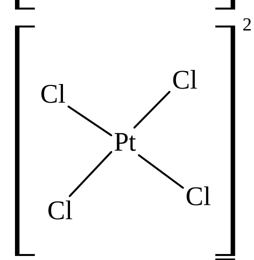
$[sp^3d]$

$[Ni(CO)_4]$


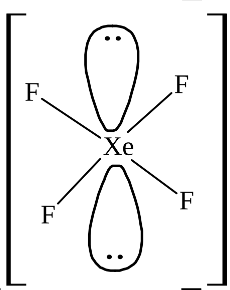
$[sp^3]$

$[XeO_2F_2]$


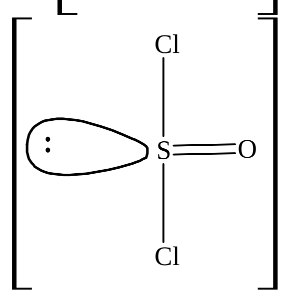
$[sp^3d]$

$[PtCl_4]^{2-}$


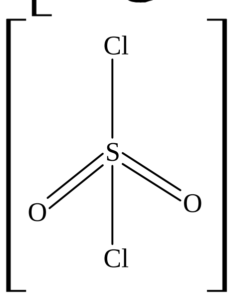
$[dsp^2]$

$[XeF_4]$


$[sp^3d^2]$

$[SOCl_2]$


$[sp^3]$

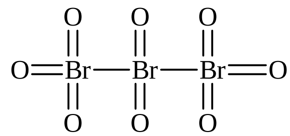
$[SO_2Cl_2]$


$[sp^3]$

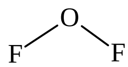
*Q.10 Consider the following molecules: Br_3O_8, F_2O, $H_2S_4O_6$, $H_2S_5O_6$, and C_3O_2.
Count the number of atoms existing in their zero oxidation state in each molecule.
Their sum is____.

Sol. **06**

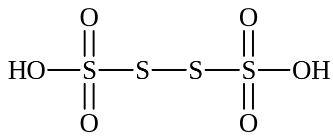
(i)



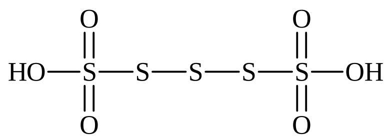
(ii)



(iii)



(iv)



(v) O=C=C=C=O

Total number of atoms having zero oxidation state = **6.**

*Q.11 For He^{+}, a transition takes place from the orbit of radius 105.8 pm to the orbit of radius 26.45 pm. The wavelength (in nm) of the emitted photon during the transition is ___.
[Use:
Bohr radius, a = 52.9 pm
Rydberg constant, $R_H = 2.2 \times 10^{-18}$ J
Planck's constant, h = 6.6×10^{-34} J s
Speed of light, c = 3×10^{8} m s^{-1}]

Sol. **30**

$$r_n = \frac{52.9 \times n^2}{Z} \text{pm}$$

$$105.8 = \frac{52.9 \times n_1^2}{2} \qquad \therefore n_1^2 = 4, \ n_1 = 2$$

$$26.45 = \frac{52.9 \times n_2^2}{2} \qquad \therefore n_2 = 1$$

$$\frac{1}{\lambda} = 109677 \times 4 \times \frac{3}{4}$$

$$\lambda = \frac{4}{109677 \times 4 \times 3} \text{cm}$$

$$= \frac{10^7}{109677 \times 3} = \frac{10^7}{329031} \text{nm}$$

λ = 30.3 nm ≈ 30 nm

Q.12 50 mL of 0.2 molal urea solution (density = 1.012 g mL^{-1} at 300 K) is mixed with 250 mL of a solution containing 0.06 g of urea. Both the solutions were prepared in the same solvent. The osmotic pressure (in Torr) of the resulting solution at 300 K is ___.
[Use: Molar mass of urea = 60 g mol^{-1}; gas constant, R = 62 L Torr K^{-1} mol^{-1} ;
Assume, $\Delta_{mix}H = 0$, $\Delta_{mix}V = 0$]

Sol. **682**

0.2 molal means 0.2 moles in 1000 g of solvent.

Volume = $\frac{M}{d}$

Mass of solution = 1012 g

Volume = $\frac{1012g}{1.012\ g\ ml^{-1}}$

V = 1000.00 ml

1000.00 ml $\longrightarrow$ 0.2 moles

50 ml of solution $= \frac{0.2}{1000} \times 50$ moles

n_{urea} = 0.01 moles

In 2nd solution:

$n_{urea} = \frac{0.06}{60} = 0.001$

Final molarity (M) = $\frac{n_1 + n_2}{V_1 + V_2} = \frac{0.01 + 0.001}{\frac{(50+250)}{1000}}$

$M = \frac{11}{300}$

$\pi = CRT$

$= \frac{11}{300} \times 62 \times 300$

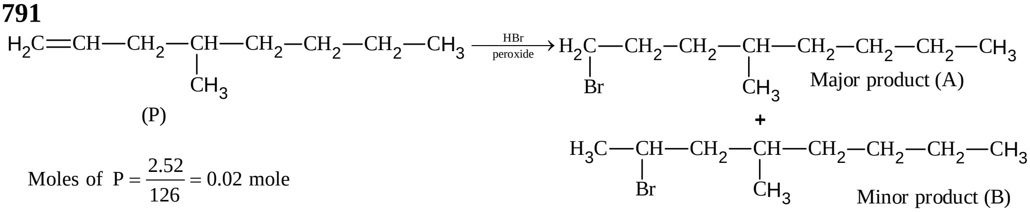
= 682 torr

Q.13 The reaction of 4-methyloct-1-ene (**P**, 2.52 g) with HBr in the presence of $(C_6H_5CO)_2O_2$ gives two isomeric bromides in a 9 : 1 ratio, with a combined yield of 50%. Of these, the entire amount of the primary alkyl bromide was reacted with an appropriate amount of diethylamine followed by treatment with aq. K_2CO_3 to give a non-ionic product **S** in 100% yield.

The mass (in mg) of **S** obtained is ___.

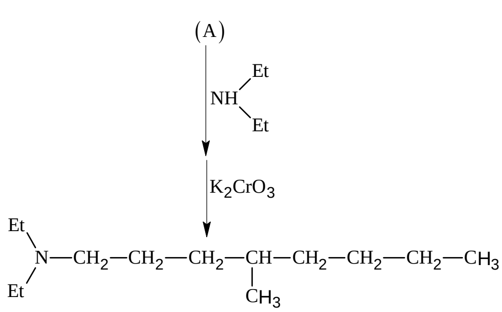
[Use molar mass (in g mol^{-1}): H = 1, C = 12, N = 14, Br = 80]

Sol. **1791**



Moles of $P = \frac{2.52}{126} = 0.02$ mole

Moles of $A = 0.02 \times \frac{9}{10} \times \frac{50}{100} = 0.009$ mole



$C_{13}H_{29}N$

$(199\ g\ mol^{-1})$

$\Rightarrow 0.009 \times 199 = 1.791$ g

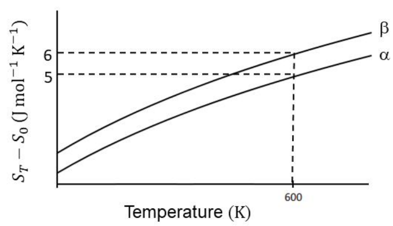
$= 1791$ mg

SECTION 4 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

"PARAGRAPH I"

The entropy versus temperature plot for phases α and β at 1 bar pressure is given.
S_T and S_0 are entropies of the phases at temperatures T and 0 K, respectively.



The transition temperature for α to β phase change is 600 K and $C_{p,\beta} - C_{p,\alpha} = 1$ J mol^{-1} K^{-1}. Assume $(C_{p,\beta} - C_{p,\alpha})$ is independent of temperature in the range of 200 to 700 K. $C_{p,\alpha}$ and $C_{p,\beta}$ are heat capacities of α and β phases, respectively.

*Q.14 The value of entropy change, $S_\beta - S_\alpha$ (in J mol^{-1} K^{-1}), at 300 K is ___.
[Use: ln2 = 0.69
Given: $S_\beta - S_\alpha = 0$ at 0 K]

Sol. **0.31**

$$S = S_0 + \int C_p \frac{dT}{T}$$

$$S_\alpha = S_0 + \int (C_p)_\alpha \frac{dT}{T}$$

$$S_\beta = S_0 + \int (C_p)_\beta \frac{dT}{T}$$

$$S_\beta - S_\alpha = \left[(C_p)_\beta - (C_p)_\alpha\right]\int \frac{dT}{T}$$

Given $C_{P_\beta} - C_{P_\alpha} = 1$

$S_\beta - S_\alpha = \ln T + C$ at any temperature T.

$$(S_\beta - S_\alpha)_{T_2} - (S_\beta - S_\alpha)_{T_1} = \ln T_2 - \ln T_1$$

T_2 = 600 K, T_1 = 300 K, from the graph S_β - S_α at 600°C = 1

(1) – $(S_\beta - S_\alpha)_{300}$ = ln 600 – ln 300

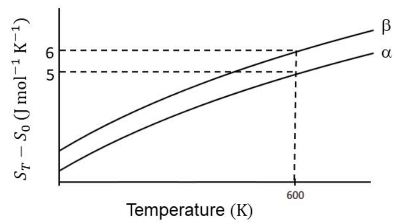
1 – $(S_\beta - S_\alpha)_{300}$ = ln 2 = 0.69

$$\Rightarrow (S_\beta - S_\alpha)_{300} = 1 - 0.69$$

= 0.31

"PARAGRAPH I"

The entropy versus temperature plot for phases α and β at 1 bar pressure is given. S_T and S_0 are entropies of the phases at temperatures T and 0 K, respectively.



The transition temperature for α to β phase change is 600 K and $C_{p,\beta} - C_{p,\alpha}$ = 1 J mol^{-1} K^{-1}. Assume $(C_{p,\beta} - C_{p,\alpha})$ is independent of temperature in the range of 200 to 700 K. $C_{p,\alpha}$ and $C_{p,\beta}$ are heat capacities of α and β phases, respectively.

*Q.15 The value of enthalpy change, $H_\beta - H_\alpha$ (in J mol^{-1}), at 300 K is ___.

Sol. **300**

Transition : $\alpha \rightleftharpoons \beta$; $\Delta G = 0$

So, $\Delta H = T\Delta S$

$\Delta H_{600} = 600 \times 1 \quad \because \Delta S = 1$

$= 600$ J mol^{-1}

From Krichoff's law

$$\Delta C_p = \frac{\Delta H_{600} - \Delta H_{300}}{600 - 300}$$

$$1 = \frac{600 - \Delta H_{300}}{300}$$

$\Delta H_{300} = 300$ J mol^{-1}

"PARAGRAPH II"

A trinitro compound, 1,3,5-tris-(4-nitrophenyl)benzene, on complete reaction with an excess of Sn/HCl gives a major product, which on treatment with an excess of $NaNO_2$/HCl at 0^oC provides **P** as the product. **P**, upon treatment with excess of H_2O at room temperature, gives the product **Q**. Bromination of **Q** in aqueous medium furnishes the product **R**. The compound **P** upon treatment with an excess of phenol under basic conditions gives the product **S**.
The molar mass difference between compounds **Q** and **R** is 474 g mol^{-1} and between compounds **P** and **S** is 172.5 g mol^{-1}.

Q.16 The number of heteroatoms present in one molecule of **R** is ______ .
[Use: Molar mass (in g mol–1): H = 1, C = 12, N = 14, O = 16, Br = 80, Cl = 35.5
Atoms other than C and H are considered as heteroatoms]

Sol. **9**

"PARAGRAPH II"

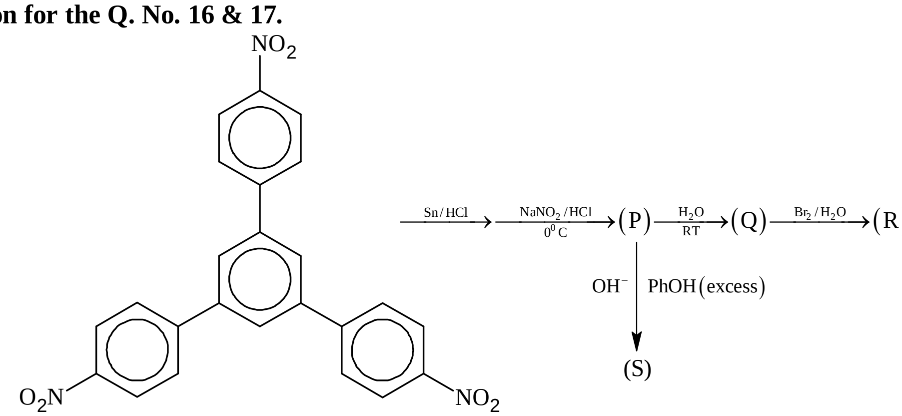
A trinitro compound, 1,3,5-tris-(4-nitrophenyl)benzene, on complete reaction with an excess of Sn/HCl gives a major product, which on treatment with an excess of $NaNO_2/HCl$ at 0^oC provides **P** as the product. **P**, upon treatment with excess of H_2O at room temperature, gives the product **Q**. Bromination of **Q** in aqueous medium furnishes the product **R**. The compound **P** upon treatment with an excess of phenol under basic conditions gives the product **S**.

The molar mass difference between compounds **Q** and **R** is 474 g mol^{-1} and between compounds **P** and **S** is 172.5 g mol^{-1}.

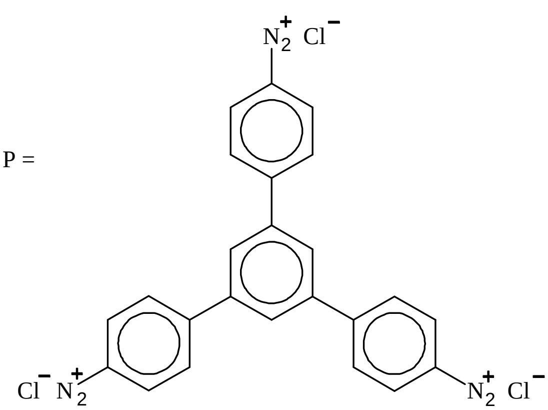
Q.17 The total number of carbon atoms and heteroatoms present in one molecule of **S** is ______ .
[Use: Molar mass (in g mol^{-1}): H = 1, C = 12, N = 14, O = 16, Br = 80, Cl = 35.5
Atoms other than C and H are considered as heteroatoms]

Sol. 51

Solution for the Q. No. 16 & 17.



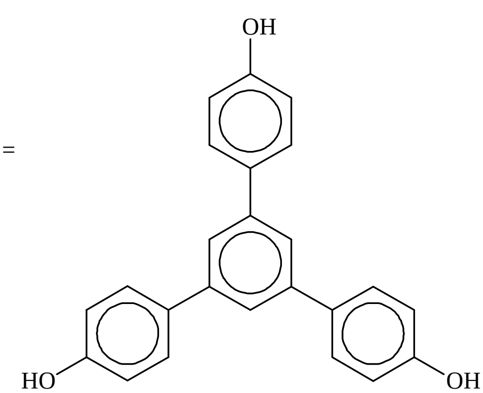
P =



$(C_{24}H_{15}N_6Cl_3)$

493.5 g mol^{-1}

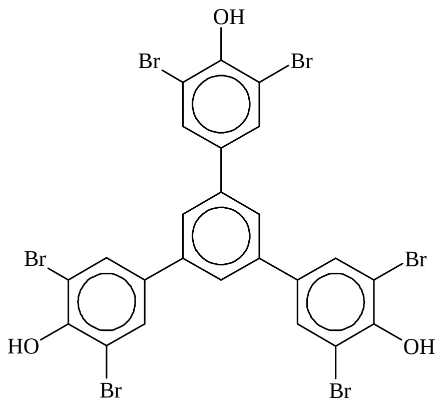
Q =



$(C_{24}H_{18}O_3)$

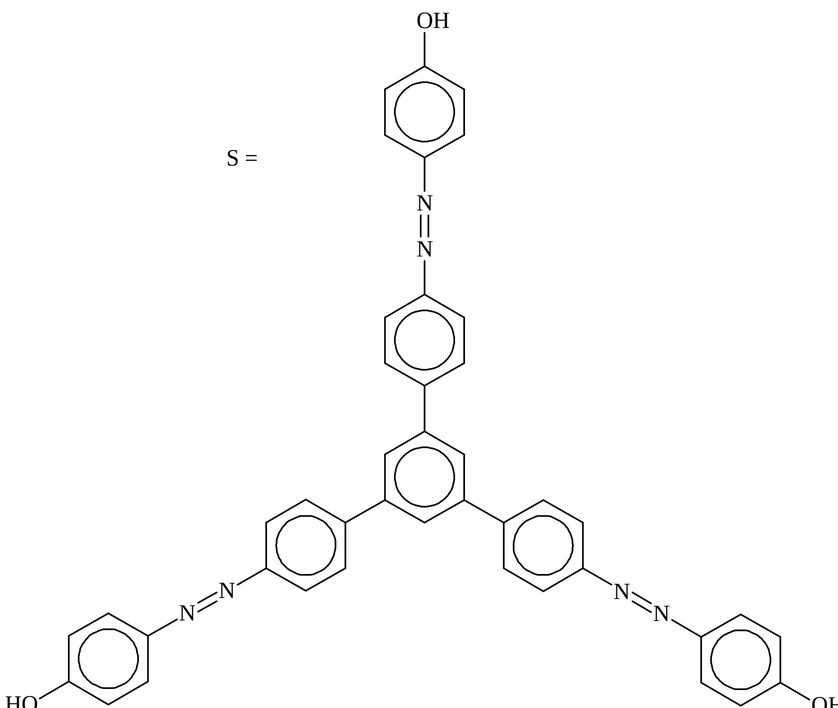
354 g mol^{-1}

R =



$(C_{24}H_{12}Br_6O_3)$
828 g mol^{-1}

S =



$(C_{42}H_{30}N_6O_3)$
666 g mol^{-1}

Note: ***For the benefit of the students, specially the aspiring ones, the question of JEE(advanced), 2023 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test. For this test the time allocated in Mathematics, Physics and Chemistry are 30 minutes, 25 minutes and 25 minutes respectively.***

SOLUTIONS TO JEE (ADVANCED) – 2023 (PAPER-1)

Mathematics

SECTION 1 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: + 4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: + 3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: + 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: + 1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: – 2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

Q.1. Let $S = (0, 1) \cup (1, 2) \cup (3, 4)$ and $T = \{0, 1, 2, 3\}$. Then which of the following statements is(are) true?
(A) There are infinitely many functions from S to T
(B) There are infinitely many strictly increasing functions from S to T
(C) The number of continuous functions from S to T is at most 120
(D) Every continuous function from S to T is differentiable

Sol. **A, C, D**
Set S has infinite elements while set T has only 4 elements, therefore it is not possible to make any strictly increasing function from set S to set T.
According to structure of domain, it is possible to make a continuous function from set S to set T and number of such possible functions is 64.
Also, every continuous function from S to T is differentiable.

There are many ways to assign a value of T to elements of domain, hence infinitely many functions will exist from set S to set T.

*Q.2. Let T_1 and T_2 be two distinct common tangents to the ellipse $E: \frac{x^2}{6} + \frac{y^2}{3} = 1$ and the parabola $P: y^2 = 12x$. Suppose that the tangent T_1 touches P and E at the points A_1 and A_2, respectively and the tangent T_2 touches P and E at the points A_4 and A_3, respectively. Then which of the following statements is(are) true?

(A) The area of the quadrilateral $A_1A_2A_3A_4$ is 35 square units
(B) The area of the quadrilateral $A_1A_2A_3A_4$ is 36 square units
(C) The tangents T_1 and T_2 meet the x -axis at the point (– 3, 0)
(D) The tangents T_1 and T_2 meet the x -axis at the point (– 6, 0)

Sol. **A, C**

$y = mx \pm \sqrt{6m^2 + 3}$ (eq. of tangent for ellipse)

$y = mx + \frac{3}{m}$ (eq. of tangent for parabola)

$\Rightarrow \frac{3}{m} = \sqrt{6m^2 + 3}$

$\Rightarrow \frac{9}{m^2} = 6m^2 + 3$

$\Rightarrow 3 = 2m^4 + m^2$

$\Rightarrow 2m^4 + m^2 - 3 = 0$

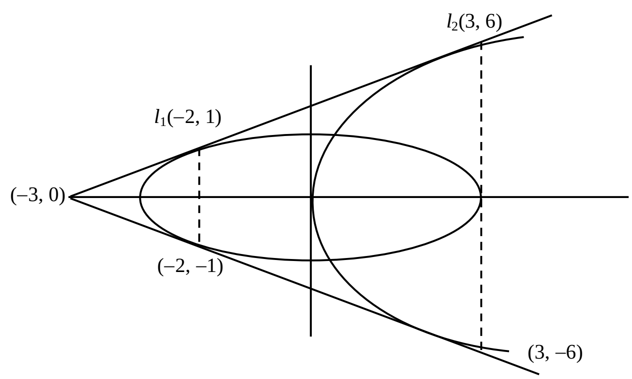
$\Rightarrow 2m^4 + 3m^2 - 2m^2 - 3 = 0$

$\Rightarrow m^2(2m^2 + 3) - 1(2m^2 + 3) = 0$

$\Rightarrow m = 1, -1$

$\Rightarrow$ Equations of tangents are $y = x + 3$ and $y = -x - 3$

$\Rightarrow$ Point of intersection = (–3, 0)



Eq. of $l_1 \rightarrow$

T = 0 (chord of contact for ellipse)

$\frac{-3x}{6} = 1$, $x = -2$

Eq. of $l_2 \rightarrow$

T = 0

$0 = 12\left(\frac{x-3}{2}\right) \Rightarrow x = 3$

$\Rightarrow$ area of quadrilateral $A_1A_2A_3A_4 = \frac{1}{2}(2 + 12) \times 5 = 35$ sq. units

Q.3. Let $f : [0, 1] \to [0, 1]$ be the function defined by $f(x) = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$. Consider the square region $S = [0, 1] \times [0, 1]$. Let $G = \{(x, y) \in S : y > f(x)\}$ be called the green region and $R = \{(x, y) \in S : y < f(x)\}$ be called the red region. Let $L_h = \{(x, h) \in S : x \in [0, 1]\}$ be the horizontal line drawn at a height $h \in [0, 1]$. Then which of the following statements is(are) true?

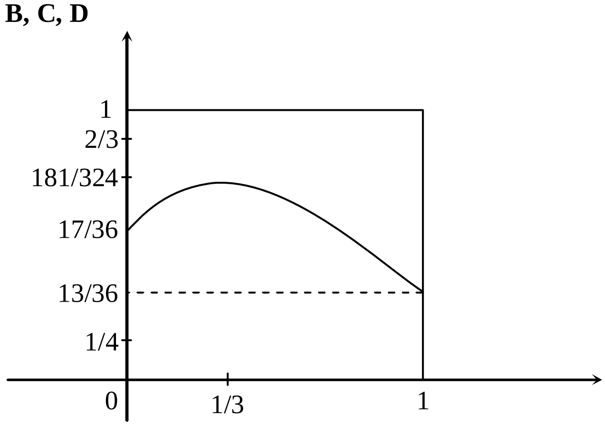
(A) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the green region below the line L_h

(B) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the red region below the line L_h

(C) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the red region below the line L_h

(D) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the green region below the line L_h

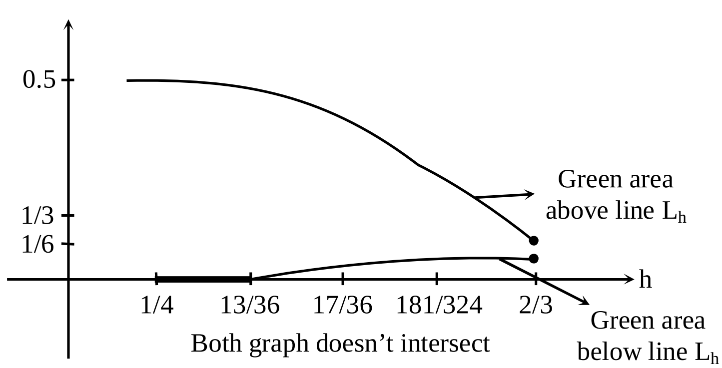
Sol. **B, C, D**



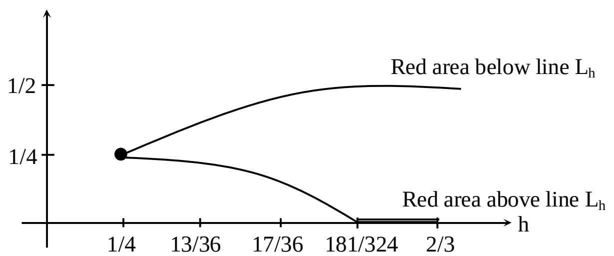
$\text{Area}_{\text{red}} = \int_0^1 f(x)\,dx = 0.5$

$\text{Area}_{\text{green}} = 0.5$

(A)

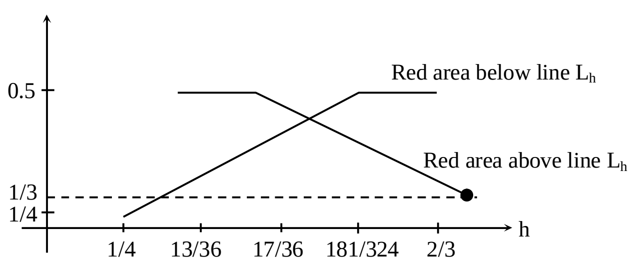


(B)



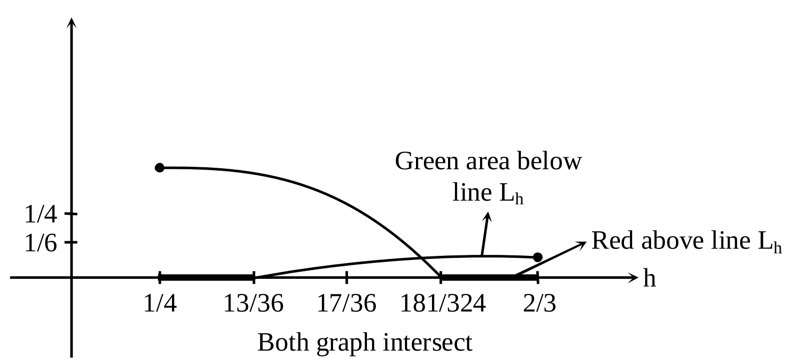
Both Graph intersect at h = $\frac{1}{4}$

(C)



Both Graph intersect.

(D)



Both graph intersect

SECTION 2 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

Q.4. Let f : (0, 1) $\rightarrow$ R be the function defined as $f(x) = \sqrt{n}$ if $x \in \left[\frac{1}{n+1}, \frac{1}{n}\right)$ where n $\in$ N. Let g : (0, 1) $\rightarrow$ R be a function such that $\int_{x^2}^{x} \sqrt{\frac{1-t}{t}}dt < g(x) < 2\sqrt{x}$ for all x $\in$ (0, 1). Then $\lim_{x\rightarrow 0} f(x)g(x)$

(A) does NOT exist
(B) is equal to 1
(C) is equal to 2
(D) is equal to 3

Sol. **C**

$$\lim_{x\to 0} f(x)g(x) = \lim_{n\to\infty} f\left(\frac{1}{n}\right)g\left(\frac{1}{n}\right)$$

$$\lim_{n\to\infty}\sqrt{n-1}\int_{1/n^2}^{1/n}\sqrt{\frac{1-t}{t}}dt \le \lim_{n\to\infty} f\left(\frac{1}{n}\right)g\left(\frac{1}{n}\right) \le \lim_{n\to\infty}\sqrt{n-1}\left(\frac{2}{\sqrt{n}}\right)$$

$$\lim_{n\to\infty}\sqrt{n-1}\int_{1/n^2}^{1/n}\sqrt{\frac{1-t}{t}}dt \le \lim_{n\to\infty} f\left(\frac{1}{n}\right)g\left(\frac{1}{n}\right) \le 2$$

$$\lim_{n\to\infty}\frac{\int_{1/n^2}^{1/n}\sqrt{\frac{1-t}{t}}dt}{\frac{1}{\sqrt{n-1}}} = \lim_{n\to\infty}\frac{-\frac{1}{n^2}\sqrt{n-1}+\frac{2}{n^3}\sqrt{n^2-1}}{-\frac{1}{2(n-1)^{3/2}}}$$

$$= \lim_{n\to\infty}\frac{2(n-1)^2}{n^2} - \frac{4(n-1)^{3/2}\sqrt{n^2-1}}{n^3} = 2$$

so $2 \le \lim_{n\to\infty} f\left(\frac{1}{n}\right)g\left(\frac{1}{n}\right) \le 2$

$\Rightarrow \lim_{n\to\infty} f\left(\frac{1}{n}\right)g\left(\frac{1}{n}\right) = 2$

Q.5. Let Q be the cube with the set of vertices $\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1, x_2, x_3 \in \{0, 1\}\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices (0, 0, 0) and (1, 1, 1) is in S. For lines ℓ_1 and ℓ_2, let $d(\ell_1, \ell_2)$ denote the shortest distance between them. Then the maximum value of $d(\ell_1, \ell_2)$ as ℓ_1 varies over F and ℓ_2 varies over S, is

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{\sqrt{8}}$

(C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{12}}$

Sol. **A**

DR's of $\overrightarrow{OG} = (1, 1, 1)$

DR's of $\overrightarrow{AC} = (-1, 1, 0)$

Equation of $\overrightarrow{OG} = \frac{x}{1} = \frac{y}{1} = \frac{z}{1}$

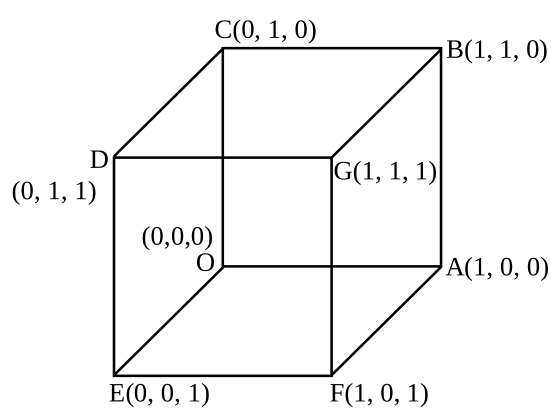
Equation of $\overrightarrow{AC} = \frac{x-1}{-1} = \frac{y}{1} = \frac{z}{0}$

$\overrightarrow{OA} = \hat{i}$

Normal of $\overrightarrow{OG}$ and $\overrightarrow{AC}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & 1 & 0 \end{vmatrix} = \left(-\hat{i}-\hat{j}+2\hat{k}\right)$$

$$\text{S.D.} = \frac{\left|\hat{i}\left(-\hat{i}-\hat{j}+2\hat{k}\right)\right|}{\left|-\hat{i}-\hat{j}+2\hat{k}\right|} = \frac{1}{\sqrt{6}}$$



Q.6. Let $X:\left\{(x,y)\in Z\times Z:\frac{x^2}{8}+\frac{y^2}{20}<1 \text{ and } y^2<5x\right\}$. Three distinct points P, Q and R are randomly chosen from X . Then the probability that P, Q and R form a triangle whose area is a positive integer, is

(A) $\frac{71}{220}$ (B) $\frac{73}{220}$

(C) $\frac{79}{220}$ (D) $\frac{83}{220}$

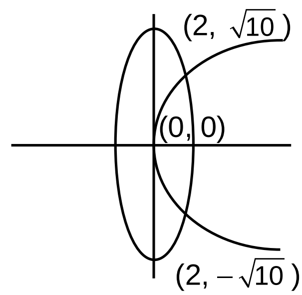
Sol. **B**

$\frac{x^2}{8}+\frac{y^2}{20}<1$ and $y^2 < 5x$

$\frac{x^2}{8}+\frac{y^2}{20}=1$ …(1)

$y^2 = 5x$ …(2)

On solving (1) and (2)



$\frac{x^2}{8}+\frac{x}{4}=1$

$x^2 + 2x = 8$

$x^2 + 2x - 8 = 0$

$x = 2, -4$

X = {(1, 1), (1, 0), (1, –1), (1, 2), (1, –2), (2, 1), (2, –1), (2, 3), (2, –3), (2, –2), (2, 2), (2, 0)}

$n(S) = {}^{12}C_3$

A is event of selecting 3 points for which area of Δ is positive integer.

$n(A) = 4 \times 7 + 9 \times 5 = 73$

$P(A) = \frac{73}{{}^{12}C_3}=\frac{73}{220}$

*Q.7. Let P be a point on the parabola $y^2 = 4ax$, where a > 0. The normal to the parabola at P meets the x-axis at a point Q. The area of the triangle PFQ, where F is the focus of the parabola, is 120. If the slope m of the normal and a are both positive integers, then the pair (a, m) is

(A) (2, 3) (B) (1, 3)

(C) (2, 4) (D) (3, 4)

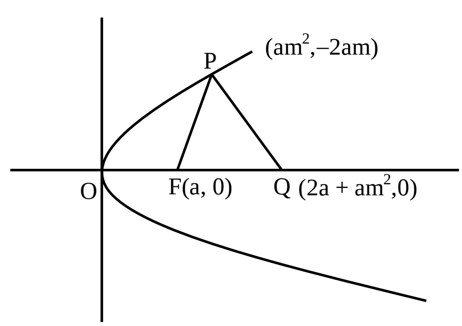
Sol. **A**

$y = mx - 2am - am^3$

$\frac{1}{2}\times|a+am^2|\times|-2am|=120$

$a^2(1 + m^2)m = 120$

(2, 3) satisfy



SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 In all other cases.

Q.8. Let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, for $x \in R$. Then the number of real solutions of the equation $\sqrt{1+\cos(2x)} = \sqrt{2}\tan^{-1}(\tan x)$ in the set $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is equal to

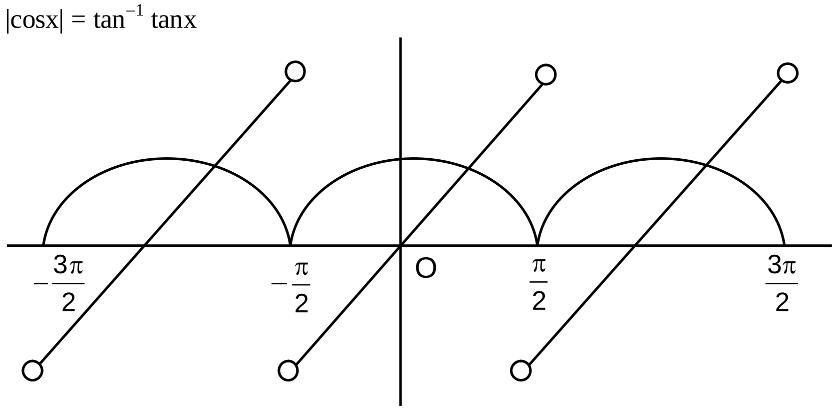
Sol. **3**

$\tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\sqrt{1+\cos 2x} = \sqrt{2}\tan^{-1}(\tan x)$

$\sqrt{2}|\cos x| = \sqrt{2}\tan^{-1}\tan x$

$|\cos x| = \tan^{-1}\tan x$



Q.9. Let $n \geq 2$ be a natural number and $f : [0, 1] \to R$ be the function defined by

$$f(x) = \begin{cases} n(1-2nx) & \text{if} \quad 0 \leq x \leq \frac{1}{2n} \\ 2n(2nx-1) & \text{if} \quad \frac{1}{2n} \leq x \leq \frac{3}{4n} \\ 4n(1-nx) & \text{if} \quad \frac{3}{4n} \leq x \leq \frac{1}{n} \\ \frac{n}{n-1}(nx-1) & \text{if} \quad \frac{1}{n} \leq x \leq 1 \end{cases}$$

If n is such that the area of the region bounded by the curves $x = 0$, $x = 1$, $y = 0$ and $y = f(x)$ is 4 , then the maximum value of the function f is

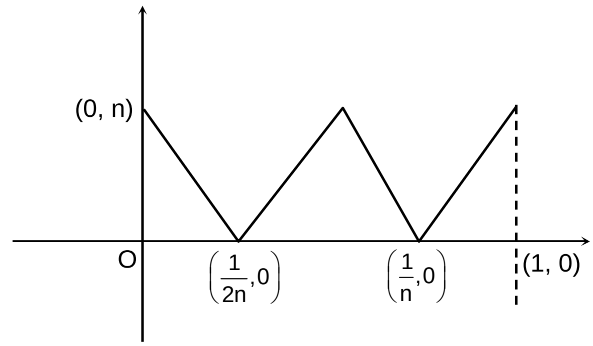
Sol. **8**

$\text{Area} = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{n-1}{2} = 4$

$= \frac{1}{2} + \frac{n-1}{2} = 4$

$\Rightarrow \quad n = 8$

So maximum value of f(x) is 8



*Q.10. Let $7\overbrace{5\cdots5}^{r}7$ denote the (r + 2) digit number where the first and the last digits are 7 and the remaining r digits are 5. Consider the sum S = 77 + 757 + 7557 + … + $7\overbrace{5\cdots5}^{98}7$. If $S = \frac{7\overbrace{5\cdots5}^{99}7 + m}{n}$, where m and n are natural numbers less than 3000, then the value of m + n is

Sol. **1219**

$T_r = 7 \times 10^{r-1} + 5(10^{r-2} + 10^{r-3} + \ldots.. + 10) + 7 \qquad r \geq 2$

$= 7 \times 10^{r-1} + 5\left(10\frac{(1-10^{r-2})}{1-10}\right) + 7$

$= 7 \times 10^{r-1} + \frac{50}{9}(10^{r-1} - 1) + 7$

$= 7 \times 10^{r-1} + \frac{50}{9}10^{r-2} + \frac{13}{9}$

$S = \sum_{r=2}^{100} T_r = \sum_{r=2}^{100}\left(7 \times 10^{r-1} + \frac{50}{9}10^{r-2} + \frac{13}{9}\right)$

$= 70\left(\frac{10^{90}-1}{10-1}\right) + \frac{50}{9} \times \left(\frac{10^{92}-1}{10-1}\right) + \frac{13}{9} \times 99$

Now, $\frac{70}{9}[10^{99} - 1] + \frac{50}{9^2}(10^{99} - 1) + 13 \times 11 = \frac{\left(7 \times 10^{100} + \frac{50}{9} \times 10^{99} + \frac{13}{9}\right) + m}{n}$

$\Rightarrow \frac{7}{9}(10^{100}) + \frac{50}{9 \times 9}10^{99} + 13 \times 11 - \frac{50}{9^2} - \frac{70}{9} = \frac{7}{n} \times 10^{100} + \frac{50}{9n} \times 10^{99} + \frac{13}{9n} + \frac{m}{n}$

n = 9

$13 \times 11 \times 9^2 - 50 - 70 \times 9 = 13 + 9m$

m = 1210

m + n = 1219

*Q.11. Let $A = \left\{\frac{1967 + 1686i\sin\theta}{7 - 3i\cos\theta} : \theta \in R\right\}$. If A contains exactly one positive integer n, then the value of n is

Sol. **281**

$\frac{281(7 + 6i\sin\theta)}{7 - 3i\cos\theta} \times \frac{7 + 3i\cos\theta}{7 + 3i\cos\theta}$

$= \dfrac{281(49 - 9\sin 2\theta)}{49 + 9\cos^2\theta} + \dfrac{562i(2\sin\theta + \cos\theta)}{49 + 9\cos^2\theta}$

for it to be positive integer (i.e. real number)

$2\sin\theta + \cos\theta = 0$

$\Rightarrow \dfrac{281(7 + 6i\sin\theta)}{7 - 3i\cos\theta} = \dfrac{281(7 - 3i\cos\theta)}{7 - 3i\cos\theta} = 281$

Q.12. Let P be the plane $\sqrt{3}x + 2y + 3z = 16$ and let $S = \left\{\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} : \alpha^2 + \beta^2 + \gamma^2 = 1\right.$ and the distance of (α, β, γ) from the plane P is $\left.\dfrac{7}{2}\right\}$. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three distinct vectors in S such that $|\vec{u} - \vec{v}| = |\vec{v} - \vec{w}| = |\vec{w} - \vec{u}|$. Let V be the volume of the parallelepiped determined by vectors, $\vec{u}, \vec{v}$ and $\vec{w}$. Then the value of $\dfrac{80}{\sqrt{3}}V$ is

Sol. **45**

$\hat{u}, \hat{v}, \hat{w}$ are equally inclined and its $\hat{u}, \hat{v}, \hat{w}$ are vertices of equilateral triangle lying on circle which is intersection of sphere $|\vec{r}| = 1$ and plane at a distance of 1/2 unit from origin & parallel to $\sqrt{3}x + 2y + 3z = 16$.

So radius of circle is $\dfrac{\sqrt{3}}{2}$ and area of triangle joining points with p.v's $\vec{u}, \vec{v}, \vec{w}$ is $\dfrac{9\sqrt{3}}{16}$. So volume of parallelepiped is $2 \times \dfrac{1}{2}\dfrac{9\sqrt{3}}{16} = \dfrac{9\sqrt{3}}{16}$, so $\dfrac{80v}{\sqrt{3}} = 45$.

*Q.13. Let a and b be two nonzero real numbers. If the coefficient of x^5 in the expansion of $\left(ax^2 + \dfrac{70}{27bx}\right)^4$ is equal to the coefficient of x^{-5} in the expansion of $\left(ax - \dfrac{1}{bx^2}\right)^7$, then the value of 2b is

Sol. **3**

General term of $\left(ax^2 + \dfrac{70}{27bx}\right)^4$ is ${}^4C_r\left(ax^2\right)^{4-r}\left(\dfrac{70}{27bx}\right)^r = {}^4C_r a^{4-r}\dfrac{(70)^r}{(27b)^r}x^{8-3r}$

for coefficient of x^5 we put $8 - 3r = 5 \Rightarrow r = 1$

$\therefore$ coeff. of x^5 is ${}^4C_1\dfrac{a^3 \times 70}{27b} = \dfrac{280}{27}\dfrac{a^3}{b}$

General term of $\left(ax - \dfrac{1}{bx^2}\right)^7$ is ${}^7C_r\dfrac{a^{7-r}(-1)^r}{b^r}x^{7-3r}$

for coeff. of x^{-5} we put $7 - 3r = -5 \Rightarrow r = 4$

$\therefore$ coeff. of x^{-5} is ${}^7C_4\dfrac{a^3}{b^4} = \dfrac{35a^3}{b^4}$

Given $\dfrac{280}{27}\dfrac{a^3}{b} = \dfrac{35a^3}{b^4} \Rightarrow b^3 = \dfrac{27}{8} \Rightarrow b = \dfrac{3}{2}$

$\Rightarrow 2b = 3$

SECTION 4 (Maximum Marks: 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

Q.14. Let α, β and γ be real numbers. Consider the following system of linear equations

$x + 2y + z = 7$

$x + \alpha z = 11$

$2x - 3y + \beta z = \gamma$

Match each entry in **List-I** to the correct entries in **List-II.**

List – I		**List – II**	
(P)	If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$, then the system has	(1)	A unique solution
(Q)	If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, then the system has	(2)	No solution
(R)	If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma \neq 28$, then the system has	(3)	Infinitely many solution
(S)	If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma = 28$, then the system has	(4)	x = 11, y = – 2 and z = 0 as a solution
		(5)	x = – 15, y = 4 and z = 0 as a solution

The correct option is:

(A) (P) → (3) (Q) → (2) (R) → (1) (S) → (4)

(B) (P) → (3) (Q) → (2) (R) → (5) (S) → (4)

(C) (P) → (2) (Q) → (1) (R) → (4) (S) → (5)

(D) (P) → (2) (Q) → (1) (R) → (1) (S) → (3)

Sol. **A**

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix} = (7\alpha - 3) - 2\beta$$

$$\Delta_x = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & \alpha \\ \gamma & -3 & \beta \end{vmatrix} = 21\alpha - 22\beta + 2\alpha\gamma - 33$$

$$\Delta_y = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & \alpha \\ 2 & \gamma & \beta \end{vmatrix} = 14\alpha + 4\beta + \gamma - \alpha\gamma - 22$$

$$\Delta_z = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma \end{vmatrix} = -2\gamma + 56$$

(P) If $\beta = \frac{1}{2}(7\alpha - 3)$ & $\gamma = 28$, then $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

So infinitely many solution

(Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ & $\gamma \neq 28$, then $\Delta = 0$ but $\Delta_z \neq 0$ so no solution.

(R) If $\beta \neq \frac{1}{2}(7\alpha - 3), \alpha = 1$ & $\gamma \neq 28$, then $\Delta \neq 0$ so unique solution.

(S) If $\beta \neq \frac{1}{2}(7\alpha - 3)$, $\alpha = 1$, $\gamma = 28$, then $\Delta \neq 0$

$\Delta = (7\alpha - 3) - 2\beta = 4 - 2\beta$

$\Delta_x = 44 - 22\beta$

$\Delta_y = 4\beta - 8$

$\Delta_z = 0$

x = 11, y = – 2, z = 0

*Q.15. Consider the given data with frequency distribution

x_i 3 8 11 10 5 4

f_i 5 2 3 2 4 4

Match each entry in **List-I** to the correct entries in **List-II.**

List – I		List – II	
(P)	The mean of the above data is	(1)	2.5
(Q)	The median of the above data is	(2)	5
(R)	The mean deviation about the mean of the above data is	(3)	6
(S)	The mean deviation about the median of the above data is	(4)	2.7
		(5)	2.4

The correct option is:

(A) (P) → (3) (Q) → (2) (R) → (4) (S) → (5)
(B) (P) → (3) (Q) → (2) (R) → (1) (S) → (5)
(C) (P) → (2) (Q) → (3) (R) → (4) (S) → (1)
(D) (P) → (3) (Q) → (3) (R) → (5) (S) → (5)

Sol. **A**

x_i	f_i	$f_i x_i$	$f_i \lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - M \rvert$
3	5	15	15	10
4	4	16	8	4
5	4	20	4	0
8	2	16	4	6
10	2	20	8	10
11	3	33	15	18
	$\Sigma f_i = 20$	$\Sigma f_i x_i = 120$	sum = 54	sum = 48

(P) Mean $(\bar{x}) = \frac{120}{20} = 6$

(Q) Median = $\frac{(10^{th} + 11^{th})\text{ observation}}{2} = 5$

(R) M.D. $(\bar{x}) = \dfrac{\Sigma f_i |x_i - \bar{x}|}{\Sigma f_i} = \dfrac{54}{20} = 2.7$

(S) M.D. (M) = $\dfrac{\Sigma f_i |x_i - M|}{\Sigma f_i} = \dfrac{48}{20} = 2.4$

Q.16. Let ℓ_1 and ℓ_2 be the lines $\vec{r}_1 = \lambda(\hat{i} + \hat{j} + \hat{k})$ and $\vec{r}_2 = (\hat{j} - \hat{k}) + \mu(\hat{i} + \hat{k})$, respectively. Let X be the set of all the planes H that contain the line ℓ_1. For a plane H, let d(H) denote the smallest possible distance between the points of ℓ_2 and H . Let H_0 be a plane in X for which $d(H_0)$ is the maximum value of d(H) as H varies over all planes in X .

Match each entry in **List-I** to the correct entries in **List-II.**

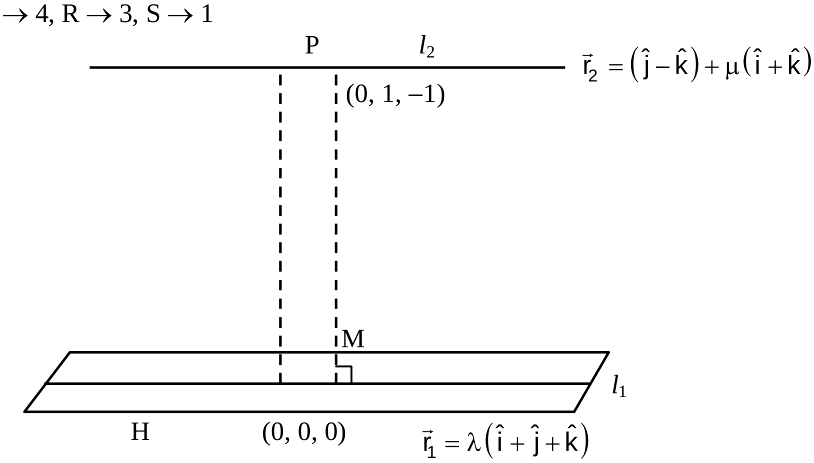
	List – I		**List – II**
(P)	The value of $d(H_0)$ is	(1)	$\sqrt{3}$
(Q)	The distance of the point (0, 1, 2) from H_0 is	(2)	$\dfrac{1}{\sqrt{3}}$
(R)	The distance of origin from H_0 is	(3)	0
(S)	The distance of origin from the point of intersection of planes y = z , x = 1 and H_0 is	(4)	$\sqrt{2}$
		(5)	$\dfrac{1}{\sqrt{2}}$

The correct option is:

(A) (P) → (2) (Q) → (4) (R) → (5) (S) → (1)
(B) (P) → (5) (Q) → (4) (R) → (3) (S) → (1)
(C) (P) → (2) (Q) → (1) (R) → (3) (S) → (2)
(D) (P) → (5) (Q) → (1) (R) → (4) (S) → (2)

Sol. **B**

P → 5, Q → 4, R → 3, S → 1



Line l_2 is parallel to plane containing l_1

Let l, m, n be D. cosine of plane H

$l + m + n = 0$ …(1)

$l + n = 0$ …(2)

$-n + m + n = 0, m = 0$

$\dfrac{l}{\sqrt{2}} = \dfrac{n}{-\sqrt{2}} = \dfrac{m}{0}$

$\frac{l}{\sqrt{2}} = \frac{m}{0} = \frac{n}{-\sqrt{2}}$

so equation of plane H_0 is

$\sqrt{2}(x-0)+0(y-0)-\sqrt{2}(z-0)=0$

$x - z = 0$

equation of plane H.

(P) $d[H_0] = PM = \frac{0+1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$

(Q) $= \left|\frac{0-2}{\sqrt{2}}\right| = \sqrt{2}$

(R) Distance of origin from $H_0 = 0$

(S) Distance of origin from the point of intersection of planes $y = z$, $x = 1$, $x = z$.

Intersection T(1, 1, 1), Distance from origin = $\sqrt{1+1+1} = \sqrt{3}$

*Q.17. Let z be a complex number satisfying $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$, where $\bar{z}$ denotes the complex conjugate of z . Let the imaginary part of z be nonzero.

Match each entry in **List-I** to the correct entries in **List-II.**

	List – I		**List – II**
(P)	$|z|^2$ is equal to	(1)	12
(Q)	$|z-\bar{z}|^2$ is equal to	(2)	4
(R)	$|z|^2 + |z+\bar{z}|^2$ is equal to	(3)	8
(S)	$|z+1|^2$ is equal to	(4)	10
		(5)	7

The correct option is:

(A) (P) → (1) (Q) → (3) (R) → (5) (S) → (4)
(B) (P) → (2) (Q) → (1) (R) → (3) (S) → (5)
(C) (P) → (2) (Q) → (4) (R) → (5) (S) → (1)
(D) (P) → (2) (Q) → (3) (R) → (5) (S) → (4)

Sol. **B**

P → 2, Q → 1, R → 3, S → 5

$|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$, imaginary part of z is non-zero

Let $z = x + iy$, $y \neq 0$

Put $|x + iy|^3 + 2(x + iy)^2 + 4(x - iy) - 8 = 0$

$\Rightarrow x = 1, y^2 = 3$

(P) $|z^2| = x^2 + y^2 = 1 + 3 = 4$

(Q) $|z-\bar{z}|^2 = |x + iy - x + iy|^2 = 4y^2 = 12$

(R) $|z|^2 + |z+\bar{z}|^2 = x^2 + y^2 + |x + iy + x - iy|^2 = x^2 + y^2 + 4x^2 = 5x^2 + y^2 = 5 + 3 = 8$

(S) $|z + 1|^2 = |x + iy + 1|^2 = |1+i\sqrt{3}+1|^2 = (4+3) = 7$

Physics

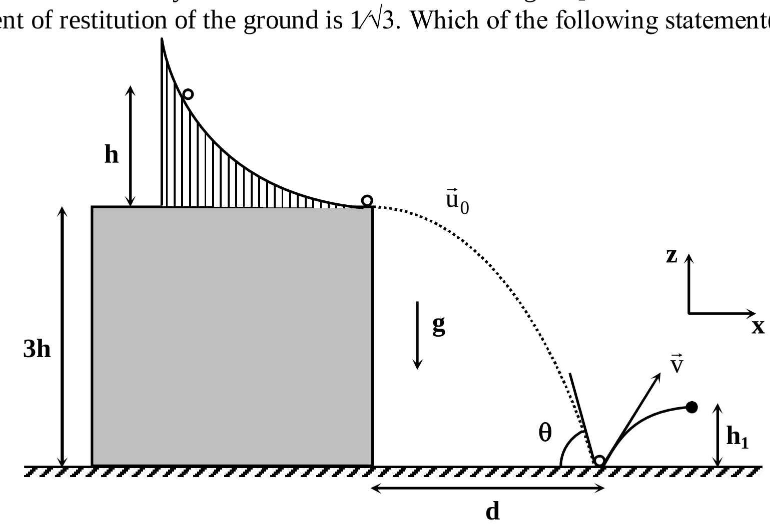
SECTION 1 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: + 4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: + 3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: + 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: + 1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: – 2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

*Q.1 A slide with a frictionless curved surface, which becomes horizontal at its lower end, is fixed on the terrace of a building of height 3 h from the ground, as shown in the figure. A spherical ball of mass m is released on the slide from rest at a height h from the top of the terrace. The ball leaves the slide with a velocity $\vec{u}_0 = u_0\hat{x}$ and falls on the ground at a distance d from the building making an angle θ with the horizontal. It bounces off with a velocity $\vec{v}$ and reaches a maximum height h_1. The acceleration due to gravity is g and the coefficient of restitution of the ground is $1/\sqrt{3}$. Which of the following statement(s) is(are) correct?



(A) $\vec{u}_0 = \sqrt{2gh}\,\hat{x}$

(B) $\vec{v} = \sqrt{2gh}(\hat{x} - \hat{z})$

(C) $\theta = 60^0$

(D) $d/h_1 = 2\sqrt{3}$

Sol. **A, C, D**

mgh = $\frac{1}{2}mu_0^2$

$u_0 = \sqrt{2gh} \Rightarrow \vec{u}_0 = \sqrt{2gh}\hat{x}$

On ground horizontal component of velocity

$v_x = \sqrt{2gh}$

Vertical component, $V_Z = \sqrt{2g \times 3h} = \sqrt{6gh}$

$\tan\theta = \frac{V_Z}{V_x} = \frac{\sqrt{6gh}}{\sqrt{2gh}} = \sqrt{3} \Rightarrow \theta = 60^0$

$\vec{v} = \sqrt{2gh}\,\hat{x} + (\sqrt{6gh})\frac{1}{\sqrt{3}}\hat{z}$

$= \sqrt{2gh}(\hat{x} + \hat{z})$

$h_1 = \frac{(eV_z)^2}{2g} = \frac{(1/3)6gh}{2g} = h$

Time to hit ground after leaving slide

$t = \sqrt{\left(\frac{6h}{g}\right)}$

$d = \left(\sqrt{2gh}\right)\left(\sqrt{\frac{6h}{g}}\right) = 2\sqrt{3}\,h$

$\frac{d}{h} = \frac{2\sqrt{3}h}{h} = 2\sqrt{3}$.

Q.2 A plane polarized blue light ray is incident on a prism such that there is no reflection from the surface of the prism. The angle of deviation of the emergent ray is $\delta = 60°$ (see Figure-1). The angle of minimum deviation for red light from the same prism is $\delta_{min} = 30°$ (see Figure-2). The refractive index of the prism material for blue light is $\sqrt{3}$. Which of the following statement(s) is(are) correct?

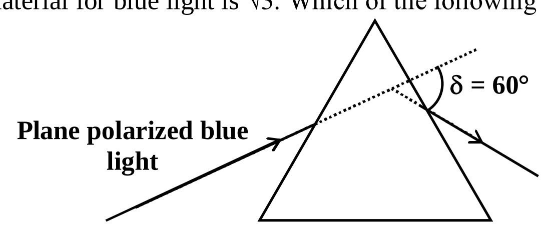


Figure-1

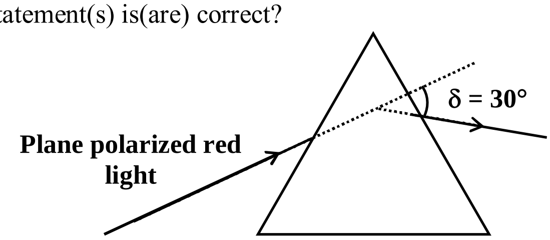


Figure-2

(A) The blue light is polarized in the plane of incidence.
(B) The angle of the prism is 45°.
(C) The refractive index of the material of the prism for red light is $\sqrt{2}$.
(D) The angle of refraction for blue light in air at the exit plane of the prism is 60^0

Sol. **A, C, D**

For Figure -1 (Blue light)

$\mu = \tan i_p$

$i_p = \tan^{-1}\sqrt{3} = 60^0$

$\delta = i + e - A \Rightarrow 60^0 = 60^0 + e - A \Rightarrow e = A$ … (i)

At incident surface, $\sin 60^0 = \sqrt{3}\sin r_1$

$\Rightarrow r_1 = 30^0$

$\because\ r_1 + r_2 = A$

$\Rightarrow\ r_2 = A - 30^0$

At emergent surface,

$$\sqrt{3}\sin(A - 30^0) = sinA$$

$$\frac{3}{2}\sin A - \frac{\sqrt{3}}{2}\cos A = \sin A$$

$\Rightarrow\ \tan A = \sqrt{3}$

$A = 60^0$

$\Rightarrow e = 60^0$

For Figure 2 (red line)

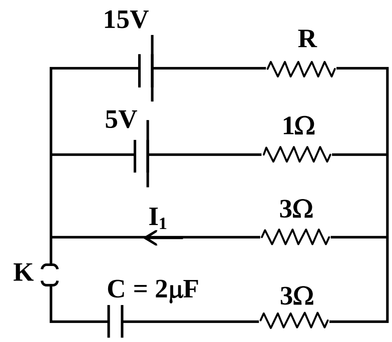
For minimum deviation

$$\frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} = \mu_R$$

$$\Rightarrow\ \frac{\sin 45^0}{\sin 30^0} = \mu_R$$

Or, $\mu_R = \sqrt{2}$

Q.3 In a circuit shown in the figure, the capacitor C is initially uncharged and the key K is open. In this condition, a current of 1 A flows through the 1 Ω resistor. The key is closed at time $t = t_0$. Which of the following statement(s) is(are) correct? [Given: $e^{-1} = 0.36$]



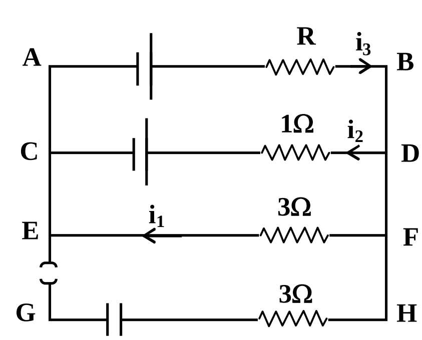
(A) The value of the resistance of R is 3 Ω

(B) For $t < t_0$, the value of current I_1 is 2A

(C) At $t = t_0 + 7.2\ \mu s$, the current in the capacitor is 0.6 A.

(D) For $t \to \infty$, the charge on the capacitor is 12 μC.

Sol. **A, B, C, D**



$$E_e = \frac{\frac{15}{R} + \frac{0}{3}}{\frac{1}{R} + \frac{1}{3}} = \frac{45}{R+3}$$

$$\Rightarrow\ i = \frac{\frac{45}{R+3} - 5}{\frac{3R}{R+3} + 1}$$

$$1 = \frac{30 - 5R}{4R + 3}$$

$R = 3\Omega$

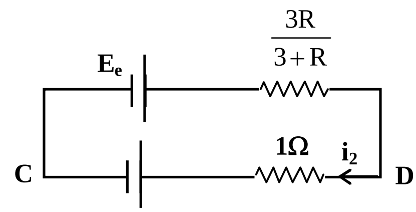
$V_{CD} = 5 + 1 \times 1 = 6$ Volt.

$\Rightarrow i_1 = \frac{6}{3} A = 2A$.

$E_e = \dfrac{\frac{15}{3}+\frac{5}{1}+\frac{0}{3}}{\frac{1}{3}+\frac{1}{1}+\frac{1}{3}} = 6$ Volts

$\frac{1}{r_e} = \frac{1}{3}+1+\frac{1}{3} = \frac{5}{3}$

$\Rightarrow r_e = \frac{3}{5}\Omega$



As $t \to \infty$, charge on capacitor,

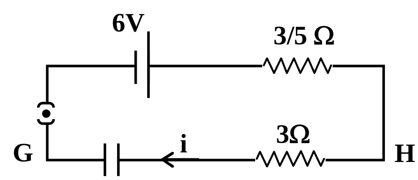
$q_0 = 2 \times 6 = 12\ \mu C$

Current through capacitor at time t is,

$i = i_0 e^{-\frac{t-t_0}{7.2\times10^{-6}}}$

At $t = t_0 + 7.2\ \mu s$

$i = \dfrac{6}{\left(\frac{3}{5}+3\right)}[e^{-1}] = 0.6A$



SECTION 2 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +3	If **ONLY** the correct option is chosen;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: –1	In all other cases.

*Q.4 A bar of mass M = 1.00 kg and length L = 0.20 m is lying on a horizontal frictionless surface. One end of the bar is pivoted at a point about which it is free to rotate. A small mass m = 0.10 kg is moving on the same horizontal surface with 5.00 ms^{-1} speed on a path perpendicular to the bar. It hits the bar at a distance L/2 from the pivoted end and returns back on the same path with speed v. After this elastic collision, the bar rotates with an angular velocity ω. Which of the following statement is correct?

(A) $\omega = 6.98$ rad $s^{-1}$and $v = 4.30\ ms^{-1}$

(B) $\omega = 3.75$ rad s^{-1} and $v = 4.30\ ms^{-1}$

(C) $\omega = 3.75$ rad s^{-1} and $v = 10.0\ ms^{-1}$

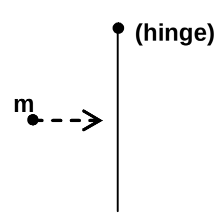
(D) $\omega = 6.80$ rad s^{-1} and $v = 4.10\ ms^{-1}$

Sol. **A**

Li = Lf

$$m\times5\times\frac{L}{2} = \frac{ML^2}{3}\times\omega - mv\times\frac{L}{2}$$

$$5 = \frac{4\omega}{3} - v$$



$v_2 - v_1 = e(u_1 - u_2)$

$$\frac{L}{2}\omega - (-v) = 1(5-0)$$

$\frac{\omega}{10} + v = 5$

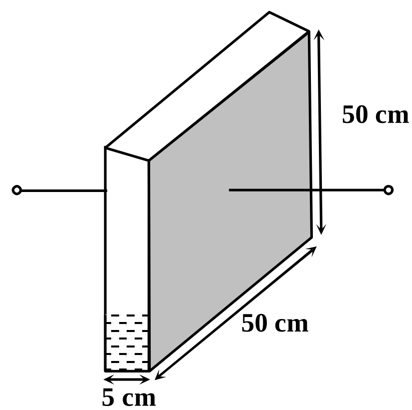
Solving (1) & (2)

$\omega = 6.98$ rad/sec

$v = 4.3$ m/s

Q.5 A container has a base of 50 cm × 5 cm and height 50 cm, as shown in the figure. It has two parallel electrically conducting walls each of area 50 cm × 50 cm. The remaining walls of the container are thin and non-conducting. The container is being filled with a liquid of dielectric constant 3 at a uniform rate of 250 $cm^3\ s^{-1}$. What is the value of the capacitance of the container after 10 seconds?

[Given: Permittivity of free space $\varepsilon_0 = 9 \times 10^{-12}\ C^2N^{-1}m^{-2}$, the effects of the non-conducting walls on the capacitance are negligible



(A) 27 pF (B) 63 pF

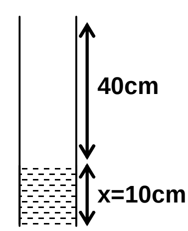
(C) 81 pF (D) 135 pF

Sol. **B**

Let container is filled upto height x in 10 sec

$250 \times 10 = 50 \times 5 \times x$

X = 10 cm



$$C = C_1 + C_2$$

$$C = \frac{A_1 \in_0}{d} + \frac{KA_2 \in_0}{d}$$

$$C = \frac{\in_0}{d}\left[A_1 + KA_2\right]$$

$$C = \frac{9\times10^{-12}}{5\times10^{-2}}\left[40\times50\times10^{-4} + 3\times50\times10\times10^{-4}\right]$$

$$= 63\times10^{-12}\,F$$

C = 63 pF

*Q.6 One mole of an ideal gas expands adiabatically from an initial state (T_A, V_0) to final state (T_f , $5V_0$). Another mole of the same gas expands isothermally from a different initial state (T_B,V_0) to the same final state (T_f , $5V_0$). The ratio of the specific heats at constant pressure and constant volume of this ideal gas is γ. What is the ratio T_A/T_B?

(A) $5^{\gamma-1}$ (B) $5^{1-\gamma}$

(C) 5^{γ} (B) $5^{1+\gamma}$

Sol. **A**

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T_A V_0^{\gamma-1} = T_f \left(5V_0\right)^{\gamma-1}$$

$$\frac{T_A}{T_f} = \left(5\right)^{\gamma-1}$$

as $T_f = T_B$

$$\therefore \frac{T_A}{T_B} = 5^{\gamma-1}$$

*Q.7 Two satellites P and Q are moving in different circular orbits around the Earth (radius R). The heights of P and Q from the Earth surface are h_P and h_Q, respectively, where $h_P = R/3$. The accelerations of P and Q due to Earth's gravity are g_P and g_Q, respectively. If $g_P/g_Q = 36/25$, what is the value of h_Q?

(A) 3R/5 (B) R/6

(C) 6R/5 (D) 5R/6

Sol. **A**

$$g_P = \frac{GM}{\left(R + \frac{R}{3}\right)^2}$$

$$g_Q = \frac{GM}{\left(R + h_Q\right)^2}$$

$$\frac{36}{25} = \frac{g_P}{g_Q} \Rightarrow \sqrt{\frac{36}{25}} = \frac{R + h_Q}{\frac{4R}{3}}$$

$$\Rightarrow \left(\frac{4R}{3}\right)\cdot\left(\frac{6}{5}\right) = R + h_Q$$

$$\Rightarrow \quad h_Q = \frac{3R}{5}$$

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If **ONLY** the correct integer is entered;

  Zero Marks : 0 In all other cases.

Q.8 A Hydrogen-like atom has atomic number Z. Photons emitted in the electronic transitions from level n = 4 to level n = 3 in these atoms are used to perform photoelectric effect experiment on a target metal. The maximum kinetic energy of the photoelectrons generated is 1. 95 eV. If the photoelectric threshold wavelength for the target metal is 310 nm, the value of Z is _______.

[Given: hc = 1240 eV-nm and Rhc = 13.6 eV, where R is the Rydberg constant, h is the Planck's constant and c is the speed of light in vacuum]

Sol. **3**

$\Delta E = E_4 - E_3$

$h\nu = 13.6z^2\left(\frac{1}{9}-\frac{1}{16}\right) = 13.6\times\frac{7}{9\times16}\left(z^2\right)$

$KE_{max} = h\nu - \phi$

$1.95 = h\nu - \frac{1240}{310}$

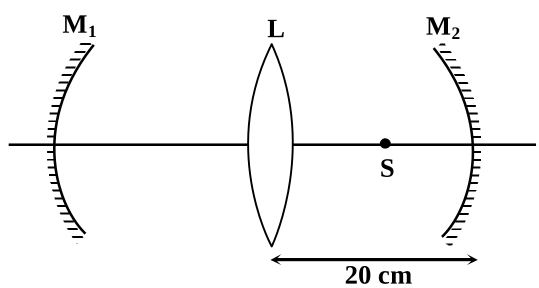
$h\nu = 1.95 + 4$

$\Rightarrow\ 5.95 = 13.6\times\frac{7}{9\times16}\left(z^2\right)$

$z^2 = 9$

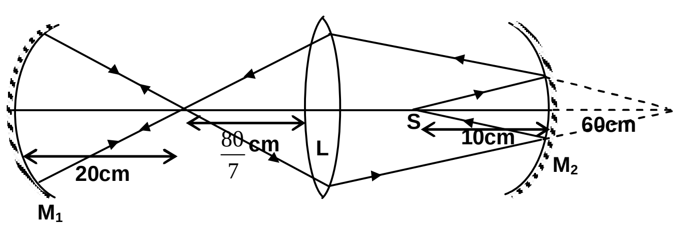
$z = 3$

Q.9 An optical arrangement consists of two concave mirrors M_1 and M_2, and a convex lens L with a common principal axis, as shown in the figure. The focal length of L is 10 cm. The radii of curvature of M_1 and M_2 are 20 cm and 24 cm, respectively. The distance between L and M_2 is 20 cm. A point object S is placed at the mid-point between L and M_2 on the axis. When the distance between L and M_1 is n/7 cm, one of the images coincides with S. The value of n is _______



Sol. **220 or 80 or 150 and you can explore other possibilities also**

Case I: If M_1 is placed at distance $\left(20+\frac{80}{7}\right)$ cm from lens, the rays retrace its path and image will be formed at S.



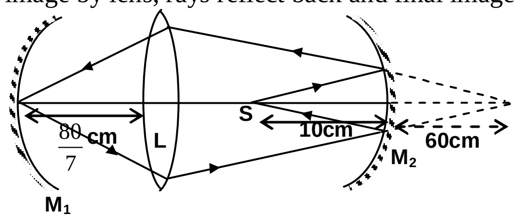
$\therefore\ n = 220$

From mirror $\frac{1}{V}+\frac{1}{-10}=\frac{1}{-12}\Rightarrow V = 60\,cm$

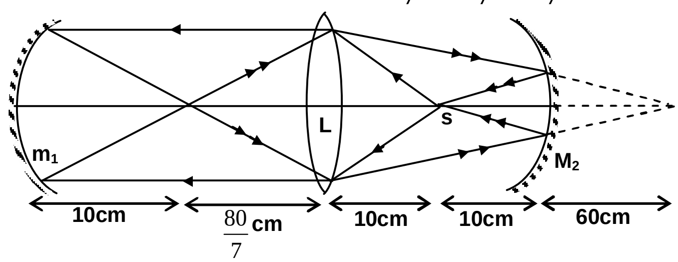
Refraction from lens $\frac{1}{V}-\frac{1}{-80}=\frac{1}{10}$

$V = \frac{80}{7}\,cm$

Case II: Consider reflection from M_2 then refraction from lens. Image is at $\frac{80}{7}\text{cm}$ left of lens if M_1 is placed at position of this image by lens, rays reflect back and final image is formed at S $\therefore$ n = 80.



Case III: First consider refraction from lens then reflection from M_1 if image due to this reflection is formed at $\frac{80}{7}\text{cm}$ left of the lens, then image after refraction with lens and reflection with M_2 will be formed at S. Then distance between L and M_1 = 10 + $\frac{80}{7} = \frac{150}{7} = \frac{n}{7} \Rightarrow n = 150$



Q.10 In an experiment for determination of the focal length of a thin convex lens, the distance of the object from the lens is 10 ± 0.1 cm and the distance of its real image from the lens is 20 ± 0.2 cm. The error in the determination of focal length of the lens is n%. The value of n is _______.

Sol. **1**

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{\Delta f}{f^2} = \pm\left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right)$$

$$\Rightarrow \frac{\Delta f}{f} = \pm\left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right)f$$

$$= \pm\left(\frac{0.2}{(20)^2} + \frac{0.1}{(10)^2}\right) \times \frac{20}{3}$$

$$\Rightarrow \frac{\Delta f}{f} = \pm 0.01$$

$$\Rightarrow \frac{\Delta f}{f} \times 100\% = \pm 1\%$$

*Q.11 A closed container contains a homogeneous mixture of two moles of an ideal monatomic gas (γ = 5/3) and one mole of an ideal diatomic gas (γ = 7/5). Here, γ is the ratio of the specific heats at constant pressure and constant volume of an ideal gas. The gas mixture does a work of 66 Joule when heated at constant pressure. The change in its internal energy is ________ Joule.

Sol. **121**

for isobaric process, work done

$W = (n_1 + n_2) R\Delta T$

$W = 3R\Delta T$

$66 = 3R\Delta T$

$R\Delta T = 22$

Change in internal energy

$$\Delta u = \frac{f_1}{2} n_1 R\Delta T + \frac{f_2}{2} n_2 R\Delta T \qquad \left[\text{Degree of freedom, } f = \frac{2}{\gamma - 1}\right]$$

$$\frac{3}{2} \times 2 \times R\Delta T + \frac{5}{2} \times 1 \times R\Delta T$$

$$\frac{11}{2} \times R\Delta T = 121 \text{ Joule}$$

*Q.12. A person of height 1.6 m is walking away from a lamp post of height 4 m along a straight path on the flat ground. The lamp post and the person are always perpendicular to the ground. If the speed of the person is 60 cm s^{-1}, The speed of the tip of the person's shadow on the ground with respect to the person is _______ cm s^{-1}

Sol. **40**

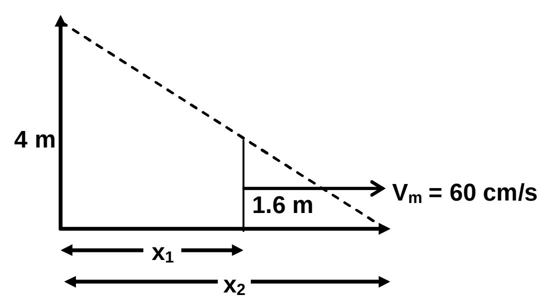
$$\frac{4}{x_2} = \frac{1.6}{(x_2 - x_1)}$$

$$\Rightarrow \quad 3x_2 = 5x_1$$

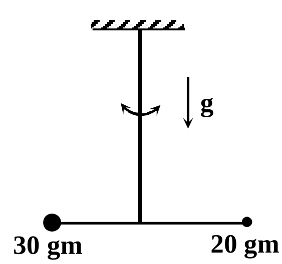
$$\Rightarrow \quad 3\frac{dx_2}{dt} = 5\frac{dx_1}{dt}$$

$$\Rightarrow \quad \frac{dx_2}{dt} = \frac{5}{3} \times 60 = 100 \text{ cm/s}$$

$\Rightarrow \quad V_{rel} = 40$ cm/sec

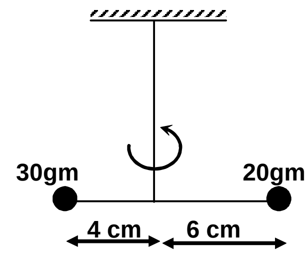


*Q.13 Two point-like objects of masses 20 gm and 30 gm are fixed at the two ends of a rigid massless rod of length 10 cm. This system is suspended vertically from a rigid ceiling using a thin wire attached to its center of mass, as shown in the figure. The resulting torsional pendulum undergoes small oscillations. The torsional constant of the wire is 1.2×10^{-8} Nm rad^{-1}. The angular frequency of the oscillations in $n \times 10^{-3}$ rad s^{-1}. The value of n is _____



Sol. **10**

Time period of oscillation



$$T = 2\pi\sqrt{\frac{I}{K}}$$

I = Moment of inertia
K = Torsional constant
moment of inertia I = 30 × 16 + 20 × 36
$I = 12 \times 10^{-5}$ kg m^2

$$T = 2\pi\sqrt{\frac{I}{K}}$$

$$= 2\pi\sqrt{\frac{12\times10^{-5}}{1.2\times10^{-8}}} = 200\pi \text{ sec}$$

$$\omega = \frac{2\pi}{T} = 10\times10^{-3} \text{ rad / s}$$

n = 10

SECTION 4 (Maximum Marks: 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 In all other cases.

Q.14 List-I shows different radioactive decay processes and List-II provides possible emitted particles. Match each entry in List-I with an appropriate entry from List-II, and choose the correct option.

	List-I		**List-II**
(P)	${}^{238}_{92}U \to {}^{234}_{91}Pa$	(1)	one α particle and one β^+ particle
(Q)	${}^{214}_{82}Pb \to {}^{210}_{82}Pb$	(2)	three β^- particles and one α particle
(R)	${}^{210}_{81}T\ell \to {}^{206}_{82}Pb$	(3)	two β^- particles and one α particle
(S)	${}^{228}_{91}Pa \to {}^{224}_{88}Ra$	(4)	one α particle and one β^- particle
		(5)	one α particle and two β^+ particles

(A) P → 4, Q →3, R→2, S→1
(B) P → 4, Q →1, R→2, S→5
(C) P → 5, Q →3, R→1, S→4
(D) P → 5, Q →1, R→3, S→2

Sol. **A**

Let x = No of α particles
& y = No of β^- particles (if y = +ve)
= No of β^+ particles (if y = –ve)
(P) 238 – 4x = 234 ⇒ x = 1 (one α particle)
and, 92 – 2x + y = 91
⇒ y = 1 (one β^- particle)
(Q) 214 – 4x = 210 ⇒ x = 1 (one α particle)

and, $82 - 2x + y = 82$
$\Rightarrow y = 2$ (two β^- particle)
(R) $210 - 4x = 206 \Rightarrow x = 1$ (one α particle)
and, $81 - 2x + y = 82$
$\Rightarrow y = 3$ (three β^- particle)
(S) $228 - 4x = 224 \Rightarrow x = 1$ (one α particle)
and, $91 - 2x + y = 88$
$\Rightarrow y = -1$ (one β^+ particle)

Q.15. Match the temperature of a black body given in List-I with an appropriate statement in List-II, and choose the correct option. [Given: Wien's constant as 2.9×10^{-3} m-K and $\frac{hc}{e} = 1.24\times10^{-6}\,V-m$]

	List-I		List-II
(P)	2000 K	(1)	The radiation at peak wavelength can lead to emission of photoelectrons from a metal of work function 4 eV
(Q)	3000 K	(2)	The radiation at peak wavelength is visible to human eye.
(R)	5000 K	(3)	The radiation at peak emission wavelength will result in the widest central maximum of a single slit diffraction
(S)	10000 K	(4)	The power emitted per unit area is 1/16 of that emitted by a blackbody at temperature 6000 K.
		(5)	The radiation at peak emission wavelength can be used to image human bones.

(A) $P \to 3, Q \to 5, R \to 2, S \to 3$
(B) $P \to 3, Q \to 2, R \to 4, S \to 1$
(C) $P \to 3, Q \to 4, R \to 2, S \to 1$
(D) $P \to 1, Q \to 2, R \to 5, S \to 3$

Sol. **C**
List-2
(1) radiation at peak $\lambda = \frac{hc}{4eV} = \frac{1.24\times10^{-6}}{4} = 0.31\times10^{-6} = 3100\,\overset{0}{A}$

$$\lambda_m T = 2.9\times10^{-3}$$

$$\lambda_m = \frac{2.9\times10^{-3}}{T} = 3100\times10^{-10}$$

$$T = \frac{2.9\times10^{7}}{3100} = 9354\,K \to 10000\text{ K}$$

(2) λ_m visible to human eye (violet to red)

(For 700 nm) $T = \frac{2.9\times10^{-3}}{7000\times10^{-10}} = \frac{29000}{7} = 4142 \to 5000\text{ K}$

(For 400 nm) $T = \frac{2.9\times10^{-3}}{4000\times10^{-10}} = 7250$

(3) widest central maximum $\Rightarrow \lambda_{max} \Rightarrow T_{min} \Rightarrow 2000$ K

(4) power per unit area $= \frac{1}{16}$ (power by block body at T = 6000 K)

$$= \frac{1}{16}\sigma(6000)^4 = \sigma T^4 \Rightarrow T = 3000\text{ K}$$

(5) $\lambda = 1\,\overset{0}{A}$

$$T = \frac{2.9\times10^{-3}}{10^{-10}} = 2.9\times10^{7}\ K$$

(p) $\rightarrow$ 3, (q) $\rightarrow$ 4, (r) $\rightarrow$ 2, (s) $\rightarrow$ 1

Q.16 A series LCR circuit is connected to a 45 sin(ωt) Volt source. The resonant angular frequency of the circuit is 10^5 rad s^{-1} and current amplitude at resonance is I_0. When the angular frequency of the source is $\omega = 8 \times 10^4$ rad s^{-1}, the current amplitude in the circuit is 0.05 I_0. If L = 50 mH, match each entry in List-I with an appropriate value from List-II and choose the correct option.

	List-I		**List-II**
(P)	I_0 in mA	(1)	44.4
(Q)	The quality factor of the circuit	(2)	18
(R)	The bandwidth of the circuit in rad s^{-1}	(3)	400
(S)	The peak power dissipated at resonance in Watt.	(4)	2250
		(5)	500

(A) P $\rightarrow$ 2, Q $\rightarrow$3, R$\rightarrow$5, S$\rightarrow$1
(B) P $\rightarrow$ 3, Q $\rightarrow$1, R$\rightarrow$4, S$\rightarrow$2
(C) P $\rightarrow$ 4, Q $\rightarrow$5, R$\rightarrow$3, S$\rightarrow$1
(D) P $\rightarrow$ 4, Q $\rightarrow$2, R$\rightarrow$1, S$\rightarrow$5

Sol. **B**

v = 45 sin (ωt)

$\omega_r = 10^5$ rad/s

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$10^5 = \frac{1}{\sqrt{50\times10^{-3}\times C}}$$

$$C = 2\times10^{-9}F$$

$$X_L = \omega L = 4000\Omega$$

$$X_C = \frac{1}{\omega C} = 6250\ \Omega$$

$X = X_C - X_L$

$$0.25\ I_0 = \frac{45}{Z}$$

R = 0.05 Z

$$R = 0.05\times\sqrt{2250^2 + R^2}$$

R = 112.6Ω

$$I_0 = \frac{45}{R} = 400\,mA$$

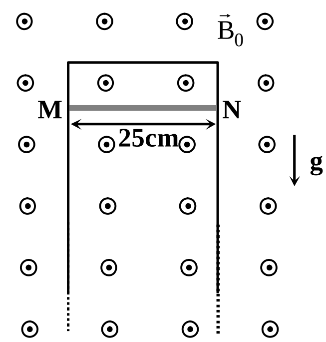
$$Q = \frac{X_L}{R} = 44.4$$

$$\text{Bandwidth} = \frac{R}{L} = 2250\ rad/s$$

$$P = \frac{V^2}{R} = 18\,W$$

(p) $\rightarrow$ 3, (q) $\rightarrow$ 1, (r) $\rightarrow$ 4, (s) $\rightarrow$ 2

Q.17 A thin conducting rod MN of mass 20 gm, length 25 cm and resistance 10 Ω is held on frictionless, long, perfectly conducting vertical rails as shown in the figure. There is a uniform magnetic field B_0 = 4 T directed perpendicular to the plane of the rod-rail arrangement. The rod is released from rest at time t = 0 and it moves down along the rails. Assume air drag is negligible. Match each quantity in List-I with an appropriate value from List-II, and choose the correct option.
[Given: The acceleration due to gravity g = 10 m s^{-2} and e^{-1}= 0.4]



	List-I		List-II
(P)	At t = 0.2 s, the magnitude of the induced emf in Volt	(1)	0.07
(Q)	At t = 0.2 s, the magnitude of the magnetic force in Newton	(2)	0.14
(R)	At t = 0.2 s, the power dissipated as heat in Watt	(3)	1.20
(S)	The magnitude of terminal velocity of the rod in m s^{-1}	(4)	0.12
		(5)	2.00

(A) P $\rightarrow$ 5, Q $\rightarrow$2, R$\rightarrow$3, S$\rightarrow$1
(B) P $\rightarrow$ 3, Q $\rightarrow$1, R$\rightarrow$4, S$\rightarrow$5
(C) P $\rightarrow$ 4, Q $\rightarrow$3, R$\rightarrow$1, S$\rightarrow$2
(D) P $\rightarrow$ 3, Q $\rightarrow$4, R$\rightarrow$2, S$\rightarrow$5

Sol. **D**

$$mg - i\ell B = ma$$

$$i = \frac{B\ell v}{R}$$

$$mg - \frac{B^2\ell^2}{R}v = \frac{mdv}{dt}$$

$$\frac{dv}{dt} = g - \frac{B^2\ell^2}{mR}v = g - cv$$

where $c = \frac{B^2\ell^2}{mR} = 5$

$$v = 2\left(1 - e^{-5t}\right)$$

at t = 0.2 $\Rightarrow$ v = 1.20
at t = 0.2 $\Rightarrow$ F_m = 0.12
P = i^2R = 0.14
V_T = 2
(p) $\rightarrow$ 3, (q) $\rightarrow$ 4, (r) $\rightarrow$ 2, (s) $\rightarrow$ 5

Chemistry

SECTION 1 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get −2 marks.

Q.1. The correct statement(s) related to processes involved in the extraction of metals is (are)
(A) Roasting of Malachite produces Cuprite.
(B) Calcination of Calamine produces Zincite.
(C) Copper pyrites is heated with silica in a reverberatory furnace to remove iron.
(D) Impure silver is treated with aqueous KCN in the presence of oxygen followed by reduction with zinc metal.

Sol. **(B, C, D)**

$$\underset{\text{(Malachite)}}{CuCO_3.Cu(OH)_2} \xrightarrow{\Delta} 2CuO + H_2O + CO_2$$

Cuprite – Cu_2O

$$\underset{\text{(Calamine)}}{ZnCO_3} \xrightarrow{\Delta} \underset{\text{(Zincite)}}{ZnO} + CO_2$$

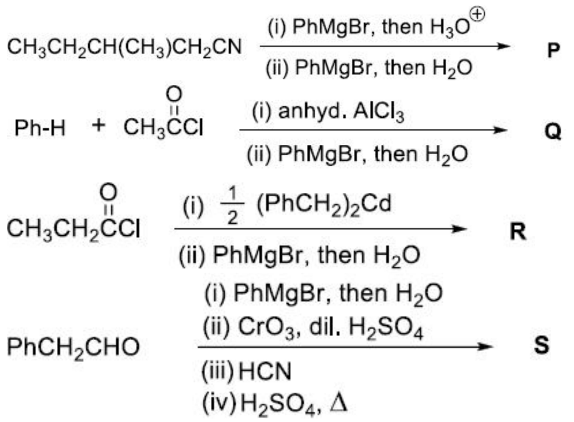
$$\underset{\text{(Copper pyrite)}}{2CuFeS_2} + \frac{11}{2}O_2 \longrightarrow Cu_2O + 2FeO + 4SO_2$$

$$FeO + SiO_2 \longrightarrow \underset{\text{Slag}}{FeSiO_3} \downarrow$$

$$\underset{\text{(Impure)}}{4Ag} + 8KCN + 2H_2O + O_2 \longrightarrow 4K\left[Ag(CN)_2\right] + 4KOH$$

$$2K\left[Ag(CN)_2\right] + Zn \longrightarrow K_2\left[Zn(CN)_4\right] + 2Ag\downarrow$$

Q.2. In the following reactions, **P**, **Q**, **R**, and **S** are the major products.



The correct statement(s) about **P**, **Q**, **R**, and **S** is (are)

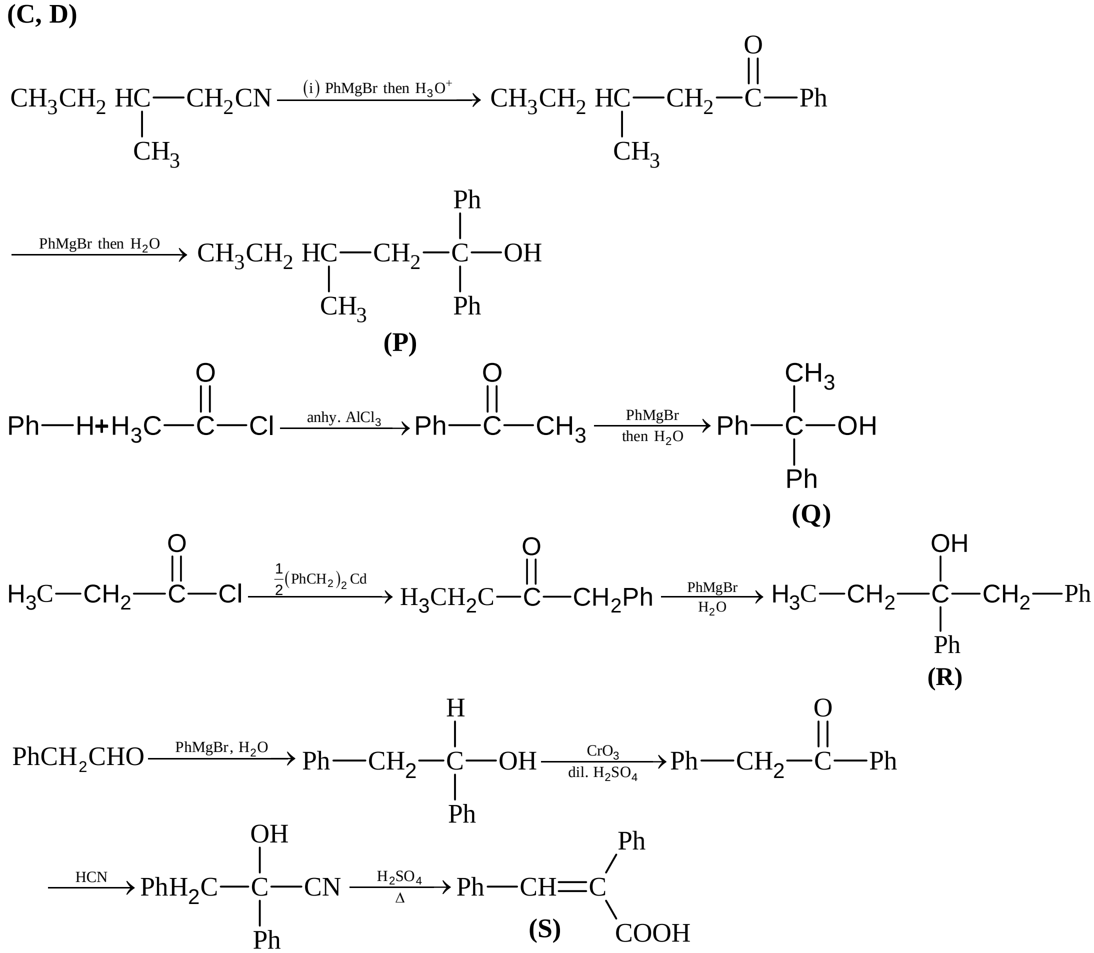
(A) Both **P** and **Q** have asymmetric carbon(s).

(B) Both **Q** and **R** have asymmetric carbon(s).

(C) Both **P** and **R** have asymmetric carbon(s).

(D) **P** has asymmetric carbon(s), **S** does **not** have any asymmetric carbon.

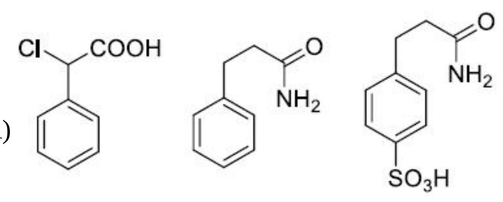
Sol. **(C, D)**



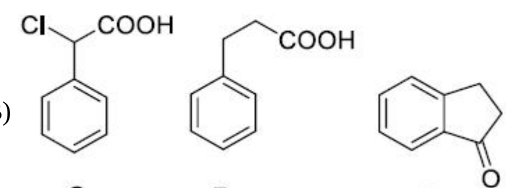
Q.3 Consider the following reaction scheme and choose the correct option(s) for the major products **Q**, **R** and **S**.

$$\text{Styrene} \xrightarrow[\text{(ii) NaOH, } H_2O_2\text{, } H_2O]{\text{(i) } B_2H_6} \text{P} \xrightarrow[\text{(ii) } Cl_2\text{, Red phosphorus (iii) } H_2O]{\text{(i) } CrO_3\text{, } H_2SO_4} \text{Q}$$

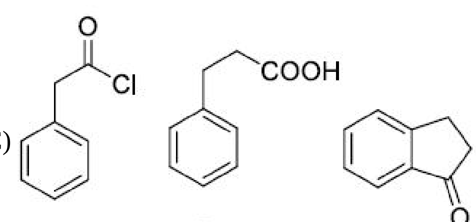
$$\text{P} \xrightarrow[\text{(ii) NaCN (iii) } H_3O^+\text{, } \Delta]{\text{(i) } SOCl_2} \text{R} \xrightarrow{\text{conc. } H_2SO_4} \text{S}$$

(A) 

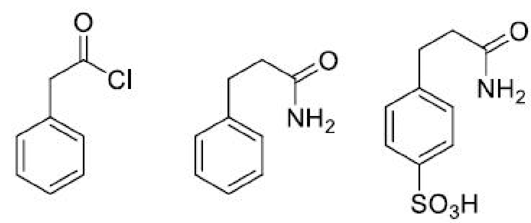
Q R S

(B) 

Q R S

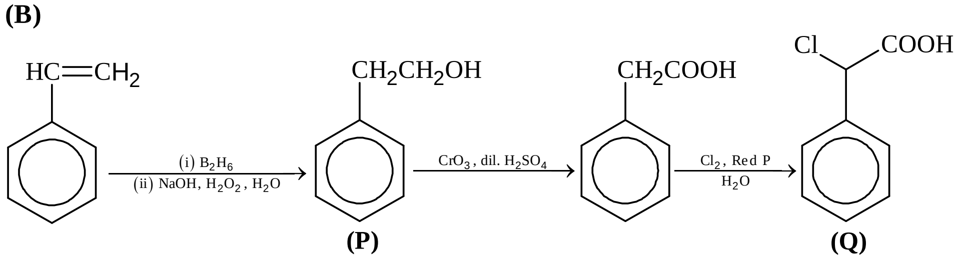
(C) 

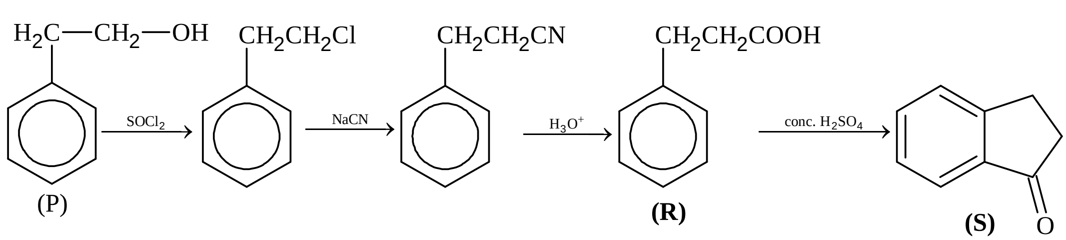
Q R S

(D) 

Q R S

Sol. **(B)**





SECTION 2 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

Q.4 In the scheme given below, **X** and **Y**, respectively, are

$$\text{Metal halide} \xrightarrow{\text{aq. NaOH}} \text{White precipitate } (\mathbf{P}) + \text{Filtrate } (\mathbf{Q})$$

$$\mathbf{P} \xrightarrow[\text{heat}]{\text{aq. } H_2SO_4,\ PbO_2 \text{ (excess)}} \mathbf{X} \text{ (a coloured species in solution)}$$

$$\mathbf{Q} \xrightarrow[\text{warm}]{MnO(OH)_2,\ \text{Conc. } H_2SO_4} \mathbf{Y} \text{ (gives blue-coloration with KI-starch paper)}$$

(A) CrO_4^{2-} and Br_2
(B) MnO_4^{2-} and Cl_2
(C) MnO_4^{-} and Cl_2
(D) $MnSO_4$ and HOCl

Sol. **(C)**

$$MnCl_2 \xrightarrow{2\,NaOH} \underset{(\text{White})\ (P)}{Mn(OH)_2}\downarrow + \underset{(Q)}{2NaCl}$$

$$\underset{(\mathbf{P})}{Mn(OH)_2} \xrightarrow[\text{aq. } H_2SO_4]{PbO_2\text{(excess)}} \underset{(X)}{HMnO_4}\ (\text{purple})$$

$$\underset{(\mathbf{Q})}{NaCl} \xrightarrow[\text{conc. } H_2SO_4\ (\text{warm})]{MnO(OH)_2} \underset{(Y)}{Cl_2}$$

Cl_2 + KI – Starch $\longrightarrow$ Blue colouration

Q.5 Plotting $1/\Lambda_m$ against $c\Lambda_m$ for aqueous solutions of a monobasic weak acid (HX) resulted in a straight line with y-axis intercept of P and slope of S. The ratio P/S is
[Λ_m = molar conductivity
Λ_m^0 = limiting molar conductivity
c = molar concentration
K_a = dissociation constant of HX]

(A) $K_a\Lambda_m^0$
(B) $K_a\Lambda_m^0/2$
(C) $2K_a\Lambda_m^0$
(D) $1/\left(K_a\Lambda_m^0\right)$

Sol. **(A)**

$$K_a = \frac{C\left(\dfrac{\lambda_m}{\lambda_m^0}\right)^2}{\left(1-\dfrac{\lambda_m}{\lambda_m^0}\right)}$$

$$K_a = \frac{C\frac{\lambda_m^2}{\lambda_m^0}}{\lambda_m^0 - \lambda_m}$$

$$K_a = \frac{C\lambda_m^2}{\lambda_m^0\left(\lambda_m^0 - \lambda_m\right)}$$

$$K_a\left(\lambda_m^0\right)^2 - K_a\lambda_m\lambda_m^0 = C\lambda_m^2$$

$$K_a\frac{\left(\lambda_m^0\right)^2}{\lambda_m} - K_a\lambda_m^0 = C\lambda_m$$

$$\frac{1}{\lambda_m} = +\left(\frac{c\lambda_m}{K_a\left(\lambda_M^0\right)^2}\right) + \frac{k_a\lambda_m^0}{K_a\left(\lambda_m^0\right)^2}$$

$$\frac{1}{\lambda_m} = \frac{c\lambda_m}{K_a\left(\lambda_M^0\right)^2} + \frac{1}{\lambda_m^0}$$

$$S = \frac{1}{K_a\left(\lambda_m^0\right)^2}; \quad P = \frac{1}{\lambda_m^0}$$

$$\frac{P}{S} = K_a\lambda_m^0$$

*Q.6 On decreasing the pH from 7 to 2, the solubility of a sparingly soluble salt (MX) of a weak acid (HX) increased from 10^{-4} mol L^{-1} to 10^{-3} mol L^{-1}. The pK_a of HX is

(A) 3 (B) 4
(C) 5 (D) 2

Sol. **(B)**

Lets assume solubility of the salt MX is S at pH = 7.

$$MX(s) \rightleftharpoons \underset{S}{M^+} + \underset{S}{X^-} \qquad K_{sp} = S^2 \qquad ...(1)$$

Lets assume solubility of the salt MX is S′ at pH = 2.

$$MX(s) \rightleftharpoons \underset{S'}{M^+} + \underset{S'}{X^-}$$

$$\underset{\substack{S' \\ \sim}}{X^-} + \underset{\substack{10^{-2} \\ 10^{-2}}}{H^+} \longrightarrow \underset{S'}{HX}$$

$$\frac{[HX]}{[X^-][H^+]} = \frac{1}{K_a}$$

$$\frac{S'}{[X^-]10^{-2}} = \frac{1}{K_a}$$

$$\frac{S'^2}{10^{-2}} = \frac{K_{sp}}{K_a} \qquad \ldots(2)$$

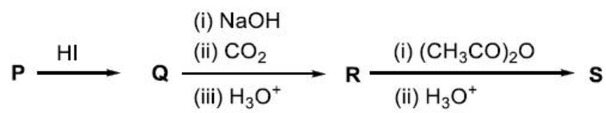
Equation (2) divided by equation (1)

$$\frac{S'^2}{S^2 \times 10^{-2}} = \frac{K_{sp}}{K_a} \times \frac{1}{K_{sp}}$$

$$\frac{\left(10^{-3}\right)}{\left(10^{-4}\right)^2 \times 10^{-2}} = \frac{1}{K_a}$$

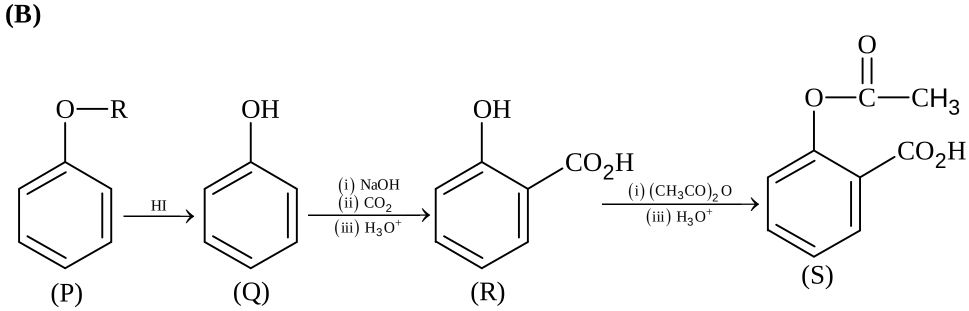
$K_a = 10^{-4}$
$pK_{a(HX)} = 4$

Q.7 In the given reaction scheme, **P** is a phenyl alkyl ether, **Q** is an aromatic compound; **R** and **S** are the major products.



The correct statement about **S** is
(A) It primarily inhibits noradrenaline degrading enzymes.
(B) It inhibits the synthesis of prostaglandin.
(C) It is a narcotic drug.
(D) It is *ortho*-acetylbenzoic acid.

Sol. **(B)**



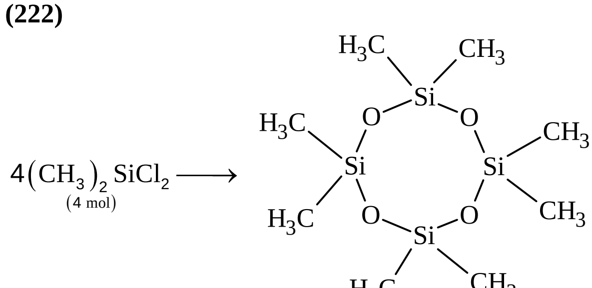
Aspirin is a non – narcotic analgesics.
It inhibits the synthesis of prostaglandin.

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 In all other cases.

*Q.8 The stoichiometric reaction of 516 g of dimethyldichlorosilane with water results in a tetrameric cyclic product **X** in 75% yield. The weight (in g) of **X** obtained is___.
[Use, molar mass (g mol^{-1}): H = 1, C = 12, O = 16, Si = 28, Cl = 35.5]

Sol. **(222)**



(Molar mass of product = 296)

4 mol $\longrightarrow$ 0.75 mol (because efficiency = 75 %)
Mass of product = 0.75 × 296 = 222 gm

*Q.9 A gas has a compressibility factor of 0.5 and a molar volume of 0.4 $dm^3\ mol^{-1}$ at a temperature of 800 K and pressure **x** atm. If it shows ideal gas behaviour at the same temperature and pressure, the molar volume will be **y** $dm^3\ mol^{-1}$. The value of **x/y** is ___.
[Use: Gas constant, R = 8×10^{-2} L atm $K^{-1}\ mol^{-1}$]

Sol. **(100)**
Z = 0.5
V = 0.4 $dm^3\ mol^{-1}$
T = 800 K
P = x

$$Z = \frac{PV}{RT} = 0.5 = \frac{x \times 0.4}{0.08 \times 800}$$

X = 80
When Z = 1, Ideal condition molar volume y dm^3

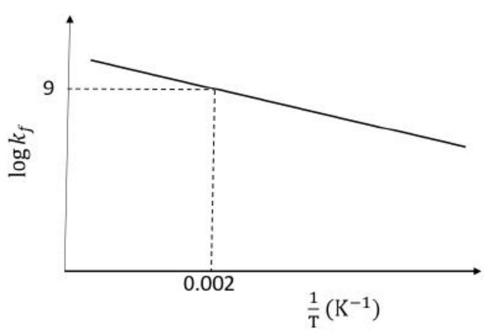
$$Z = \frac{PV}{RT}$$

$$1 = \frac{80 \times y}{0.08 \times 800}$$

y = 0.8

$$\frac{x}{y} = \frac{80}{0.8} = 100$$

*Q.10 The plot of log k_f versus 1/T for a reversible reaction $A(g) \rightleftharpoons P(g)$ is shown



Pre-exponential factors for the forward and backward reactions are $10^{15}\ s^{-1}$ and $10^{11}\ s^{-1}$, respectively. If the value of log K for the reaction at 500 K is 6, the value of | log k_b | at 250 K is ___.
[K = equilibrium constant of the reaction
k_f = rate constant of forward reaction
k_b = rate constant of backward reaction]

Sol. **(5)**

$$\log k_f = 9 \text{ at } 500\text{ K}, \therefore k_f = 10^9$$

$$\log k_b = \frac{-(E_a)_b}{2.303R}\frac{1}{T} + \log A_b$$

$$K_{eq} = 10^6 = \frac{10^9}{k_b}$$

$$k_b = 10^3$$

$$3 = \frac{-(E_a)_b}{2.303R} \times 0.02 + 11$$

$$\frac{(E_a)_b}{2.303R} = \frac{8}{0.002} = 4000$$

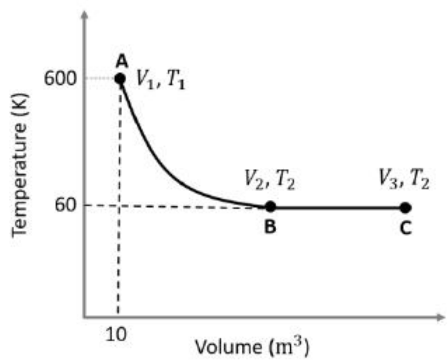
$$\log k_b = -4000 \times \frac{1}{250} + 11$$

$$= -16 + 11$$

$$= -5$$

$$|\log k_b| = 5$$

*Q.11 One mole of an ideal monoatomic gas undergoes two reversible processes ($A \rightarrow B$ and $B \rightarrow C$) as shown in the given figure:



$A \rightarrow B$ is an adiabatic process. If the total heat absorbed in the entire process ($A \rightarrow B$ and $B \rightarrow C$) is RT_2 ln 10, the value of 2 log V_3 is ___.

[Use, molar heat capacity of the gas at constant pressure, $C_{p,m} = \frac{5}{2}R$]

Sol. **(7)**

$A \rightarrow B$ (reversible adiabatic), $\gamma = \frac{5}{3}$

$$q_{total} = E_{AB} + q_{BC} = 0 + q_{BC} = q_{BC}$$

$$q_{BC} = RT_2 \ell n 10$$

For the process A to B

$$T_1 \times V_1^{\gamma-1} = T_2 \times V_2^{\gamma-1}$$

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_1}{T_2} = \frac{600}{60} = 10$$

$$\left(\frac{V_2}{V_1}\right)^{\frac{5}{3}-1} = 10$$

$$\frac{V_2}{V_1} = (10)^{\frac{3}{2}}$$

$$V_2 = 10 \times (10)^{\frac{3}{2}} = (10)^{\frac{5}{2}}$$

$$V_2 = (10)^{\frac{5}{2}}$$

$$-RT_2 \ell n 10 = -RT_2 \ell n \frac{V_3}{V_2}$$

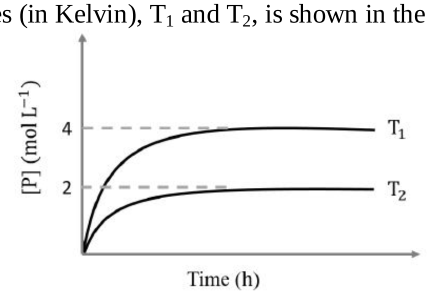
$$10 = \frac{V_3}{V_2}$$

$$V_3 = 10V_2 = 10\times(10)^{\frac{5}{2}} = (10)^{\frac{7}{2}}$$

$$2\ln V_3 = 2\times \ln 10^{\frac{7}{2}}$$

$$2\log V_3 = 2\times\frac{7}{2}\times 1 = 7$$

*Q.12 In a one-litre flask, 6 moles of A undergoes the reaction A (g) $\rightleftharpoons$ P (g). The progress of product formation at two temperatures (in Kelvin), T_1 and T_2, is shown in the figure:



If $T_1 = 2T_2$ and $\left(\Delta G_2^\theta - \Delta G_1^\theta\right) = RT_2 \ln x$, then the value of x is ……

[[ΔG_1^θ and ΔG_2^θ are standard Gibb's free energy change for the reaction at temperatures T_1 and T_2, respectively.]

Sol. **(8)**

	$A(g) \rightleftharpoons$	$P(g)$	
Initial mole	6	0	
at eq. at T_1	2	4	$\left(K_{eq}\right)_{T_1} = \frac{4}{2} = 2$
at eq. T_2	4	2	$\left(K_{eq}\right)_{T_2} = \frac{2}{4} = \frac{1}{2}$

$$\Delta G_1^o = -RT_1\ell n2$$

$$\Delta G_2^o = -RT_2\ell n\frac{1}{2} = RT_2\ell n2$$

$$\Delta G_2^o - \Delta G_1^o = RT_2\ell n2 + RT_1\ell n2 \qquad \therefore T_1 = 2T_2$$

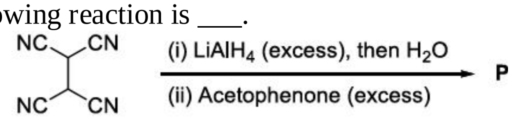
$$\Delta G_2^o - \Delta G_1^o = RT_2\ell n2 + 2RT_2\ell n2$$

$$= 3RT_2\ell n2 = RT_2\ell n8$$

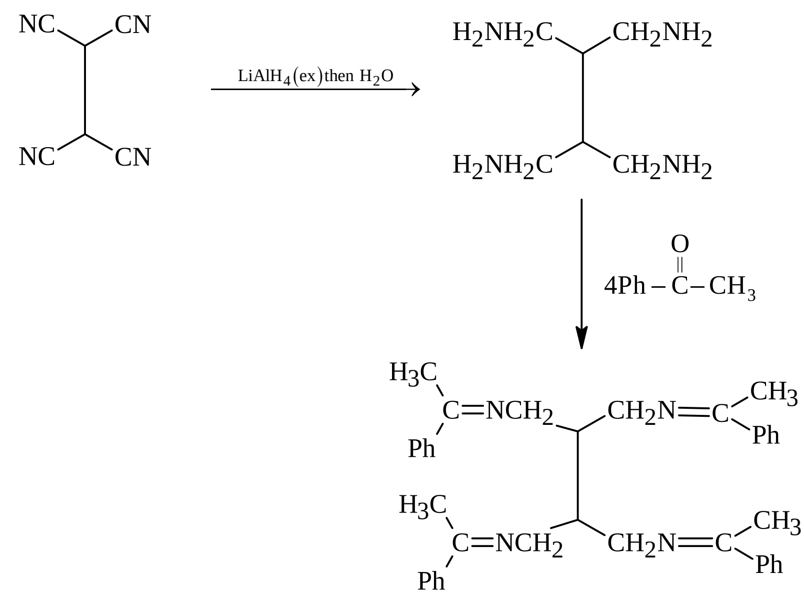
$$RT_2\ell n8 = RT_2\ell nX$$

So, X = 8

Q.13 The total number of sp^2 hybridised carbon atoms in the major product **P** (a non-heterocyclic compound) of the following reaction is ___.



Sol. **(28)**



SECTION 4 (Maximum Marks: 12)
• This section contains **FOUR (04)** Matching List Sets. • Each set has **ONE** Multiple Choice Question. • Each set has **TWO** lists: **List-I** and **List-II**. • **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5). • **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question. • Answer to each question will be evaluated according to the following marking scheme: *Full Marks* : +3 **ONLY** if the option corresponding to the correct combination is chosen; *Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered); *Negative Marks* : −1 In all other cases.

Q.14 Match the reactions (in the given stoichiometry of the reactants) in List-I with one of their products given in List-II and choose the correct option.

List-I	List-II
(P) $P_2O_3 + 3H_2O \rightarrow$	(1) $P(O)(OCH_3)Cl_2$
(Q) $P_4 + 3NaOH + 3H_2O \rightarrow$	(2) H_3PO_3
(R) $PCl_5 + CH_3COOH \rightarrow$	(3) PH_3
(S) $H_3PO_2 + 2H_2O + 4AgNO_3 \rightarrow$	(4) $POCl_3$
	(5) H_3PO_4

(A) $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 5$
(B) $P \rightarrow 3; Q \rightarrow 5; R \rightarrow 4; S \rightarrow 2$
(C) $P \rightarrow 5; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 3$
(D) $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 4; S \rightarrow 5$

Sol. **(D)**

$$P_2O_3 + 3H_2O \longrightarrow 2H_3PO_3$$

$$P_4 + 3NaOH + 3H_2O \longrightarrow PH_3 + 3NaH_2PO_2$$

$$PCl_5 + CH_3COOH \longrightarrow CH_3COCl + POCl_3 + HCl$$

$$H_3PO_2 + 2H_2O + 4AgNO_3 \longrightarrow H_3PO_4 + 4Ag + 4HNO_3$$

$P \rightarrow 2,\ Q \rightarrow 3,\ R \rightarrow 4,\ S \rightarrow 5$

Q.15 Match the electronic configurations in List-I with appropriate metal complex ions in List-II and choose the correct option.

[Atomic Number: Fe = 26, Mn = 25, Co = 27]

List-I	List-II
(P) $t_{2g}^{6}e_{g}^{0}$	(1) $[Fe(H_2O)_6]^{2+}$
(Q) $t_{2g}^{3}e_{g}^{2}$	(2) $[Mn(H_2O)_6]^{2+}$
(R) $e^{2}t_{2}^{3}$	(3) $[Co(NH_3)_6]^{3+}$
(S) $t_{2g}^{4}e_{g}^{2}$	(4) $[FeCl_4]^{-}$
	(5) $\left[CoCl_4\right]^{2-}$

(A) $P \rightarrow 1; Q \rightarrow 4; R \rightarrow 2; S \rightarrow 3$

(B) $P \rightarrow 1; Q \rightarrow 2; R \rightarrow 4; S \rightarrow 5$

(C) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 5; S \rightarrow 1$

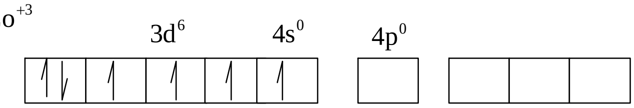
(D) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 4; S \rightarrow 1$

Sol. **(D)**

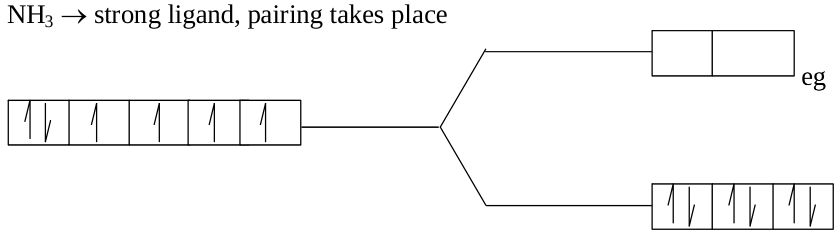
$P \rightarrow t_{2g}^{6}eg^{0}$

$\left[Co\left(NH_5\right)_6\right]^{+3}$

Co^{+3}



$NH_3 \rightarrow$ strong ligand, pairing takes place

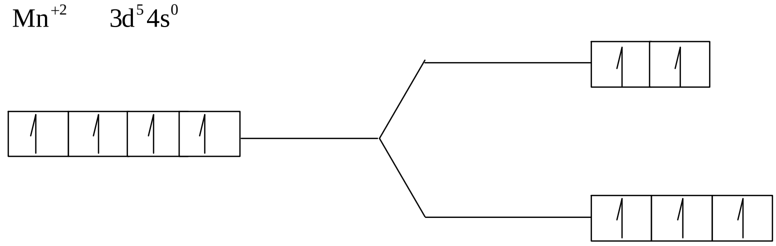


P – 3

$Q \rightarrow t_{2g}^{3}e_{g}^{2}$

$\left[Mn\left(H_2O\right)_6\right]^{+2}$, H_2O is weak ligand

Mn^{+2} $3d^{5}4s^{0}$

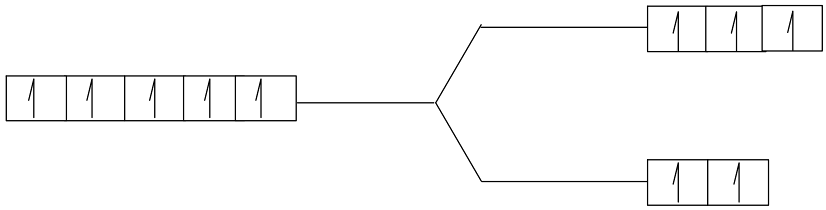


Q → 2

$R \rightarrow e_{g}^{2}.t_{2g}^{3}$

$\left[FeCl_4\right]^{-}$

$Fe^{+3} = 3d^{5}$

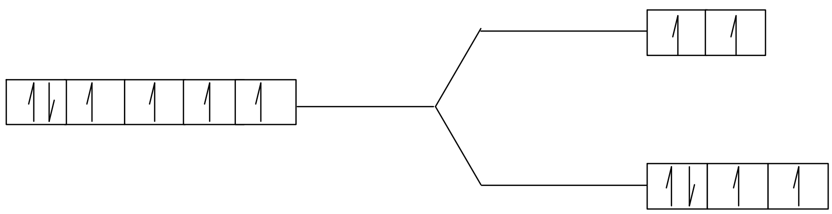


$R \rightarrow 4$

$S \rightarrow t_{2g}^{4} e_{g}^{2}$

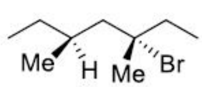
$\left[Fe\left(H_2O\right)_6\right]^{+2}$

$Fe^{+2} = 3d^6$



$S \rightarrow 1$

Q.16 Match the reactions in List-I with the features of their products in List-II and choose the correct option.

List-I	List-II
(P) (-)-1-Bromo-2-ethylpentane (single enantiomer) $\xrightarrow[S_N2 \text{ reaction}]{\text{aq. NaOH}}$	(1) Inversion of configuration
(Q) (-)-2-Bromopentane (single enantiomer) $\xrightarrow[S_N2 \text{ reaction}]{\text{aq. NaOH}}$	(2) Retention of configuration
(R) (-)-3-Bromo-3-methylhexane (single enantiomer) $\xrightarrow[S_N1 \text{ reaction}]{\text{aq. NaOH}}$	(3) Mixture of enantiomers
(S)  (single enantiomer) $\xrightarrow[S_N1 \text{ reaction}]{\text{aq. NaOH}}$	(4) Mixture of structural isomers
	(5) Mixture of diastereomers

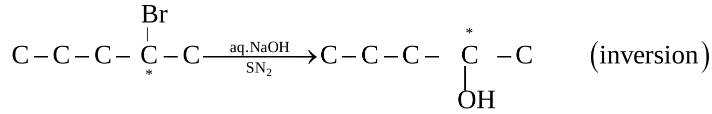
(A) $P \rightarrow 1; Q \rightarrow 2; R \rightarrow 5; S \rightarrow 3$
(B) $P \rightarrow 2; Q \rightarrow 1; R \rightarrow 3; S \rightarrow 5$
(C) $P \rightarrow 1; Q \rightarrow 2; R \rightarrow 5; S \rightarrow 4$
(D) $P \rightarrow 2; Q \rightarrow 4; R \rightarrow 3; S \rightarrow 5$

Sol. **(B)**

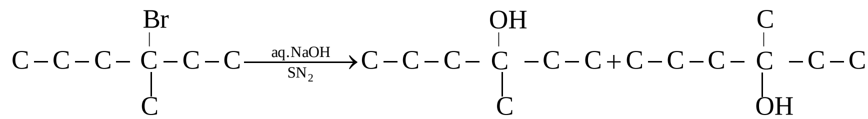
$P \rightarrow 2$

$$C-C-C-\overset{*}{\underset{\underset{C-C}{|}}{C}}-C-Br \xrightarrow[SN_2]{aq.NaOH} C-C-C-\overset{*}{\underset{\underset{C-C}{|}}{C}}-C-OH \left(\text{Retention}\right)$$

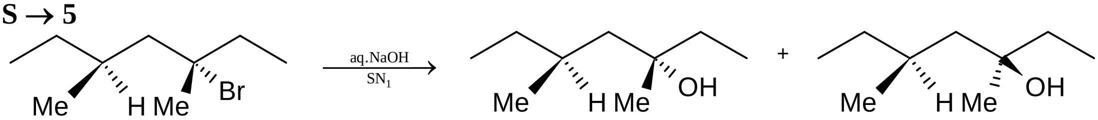
Q → 1



R → 3



S → 5



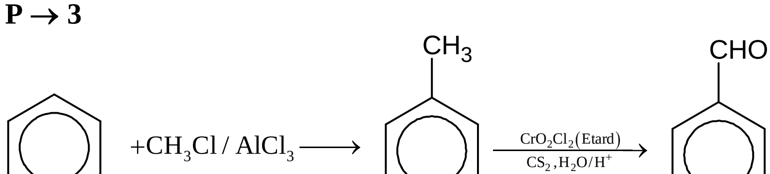
Q.17 The major products obtained from the reactions in List-II are the reactants for the named reactions mentioned in List-I. Match List-I with List-II and choose the correct option.

List-I	List-II
(P) Etard reaction	(1) Acetophenone $\xrightarrow{\text{Zn-Hg, HCl}}$
(Q) Gattermann reaction	(2) Toluene $\xrightarrow[\text{(ii) SOCl}_2]{\text{(i) KMnO}_4\text{, KOH, }\Delta}$
(R) Gattermann-Koch reaction	(3) Benzene $\xrightarrow[\text{anhyd. AlCl}_3]{\text{CH}_3\text{Cl}}$
(S) Rosenmund reduction	(4) Aniline $\xrightarrow[\text{273-278 K}]{\text{NaNO}_2\text{/HCl}}$
	(5) Phenol $\xrightarrow{\text{Zn, }\Delta}$

(A) P → 2; Q → 4; R → 1; S → 3
(B) P → 1; Q → 3; R → 5; S → 2
(C) P → 3; Q → 2; R → 1; S → 4
(D) P → 3; Q → 4; R → 5; S → 2

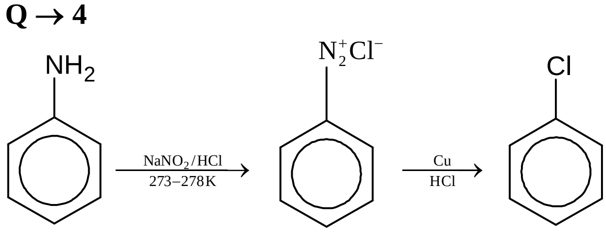
17. **(D)**

P → 3



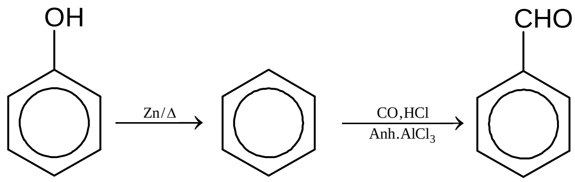
(Etard reaction)

Q → 4



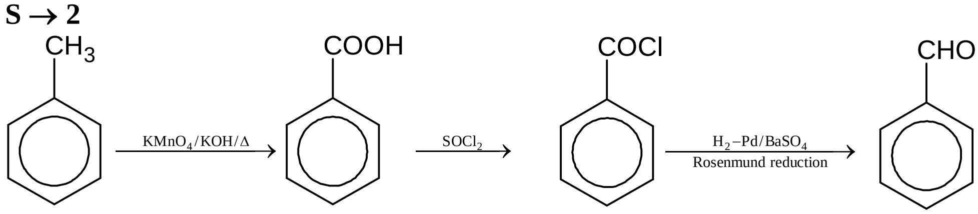
(Gatterman reaction)

R → 5



(Gattermann Koch reaction)

S → 2



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